Cyclic anelasticity of metals

V. Chmelko

Slovak University of Technology in Bratislava, Faculty of Mechanical Engineering, Institute of Applied Mechanics and Mechatronics, Námestie slobody 17, 812 31 Bratislava, Slovak Republic

Abstract

The mechanics of a solid body is based on Hooke's law. The non-linearity of the stress-strain curve in cyclic loading begins at much lower stress values than at the static yield stress point. A sensitive measurement of hysteresis loops led to discovering the anelastic strain processes below the yield stress and also below the fatigue limit. In this paper, the anelasticity limit is defined – the limit between reversible and irreversible strains – which is in compliance with the fatigue damage cumulation.

K e y   w o r d s: anelasticity, cyclic strain, fatigue

1. Introduction

Most crystalline materials demonstrate their non-linear behaviour from the lowest levels of stress loading [1–3]. At low levels of stress, the main acting deformation mechanisms are diffusion creeping, slip along the grain borders, and dislocation creeping. These processes are influenced by the grain size and temperature. A certain role in these deformation processes is played also by defects in the crystalline lattice. In the literature, the set of reasons for this anelastic behaviour of materials under stress loading is often called internal friction, and several models exist which describe this phenomenon [4–10]. Generally speaking, Hooke's law (1), according to which the deformation of the body is proportional to the stress generated by external forces on the body, is a linear approximation of the actual relation of stress and strain also in the area of stresses which do not cause a permanent deformation. This fact, or its manifestation, is in the literature denoted as anelasticity of materials. The stress-strain curve of the material in the case of a static isotropic tensile strength is described by the linear Hooke's law in the form:

$$\sigma = E\varepsilon. \tag{1}$$

The yield point which is conventionally determined by a tensile diagram as the stress at which the plastic strain reaches the value of 0.2 %, is considered as a limit between the elastic and plastic stress-deformation state of the material. Strains below this stress value are considered to be flexible. This simplification of the tensile curve of the material (since the limit of the Hooke's law validity is in fact considered the proportionality limit) is acceptable at static loading. Several above sources mention the reversibility of these anelastic processes. As the anelasticity of the stress and strain relation occurs always at the side of bigger deformations (on the right side of the line of the linear Hooke's law), it is necessary to consider the influence of the anelastic behaviour of metals on the cumulation of hysteresis energy at cyclic loading. This paper deals with the problem of acceptability of Hooke's law at the point of cyclic loading of a material.

2. Problem analysis

The yield stress applied in the form of a harmonic tensile stress amplitude on a material specimen leads to a number of cycles until the point of fracture, of an order of magnitude up to 10^4 (Table 1).

From the point of the cyclic loading, this is an area in which the overall strain dominates its plastic strain component, and for the fatigue lifetime, the decisive influence have the Manson-Coffin curve constants (ε'_f is fatigue ductility coefficient, c is fatigue

*Corresponding author: tel.: +421 2 57296225; e-mail address: vladimir.chmelko@stuba.sk

T a b l e 1. Examples of the cyclic properties of metallic materials (σ_f' is fatigue strength coefficient, b is fatigue strength exponent, ε_f' is fatigue ductility coefficient, c is fatigue ductility exponent) measured by the electrohydraulic pulsator on the same specimens as shown in Fig. 4

	Material σ_{kt} (MPa)	Yield stress N_f	Number of cycles until the fracture N_T	Transitive number of cycles σ_a (MPa)	Stress amplitude for N_T $\sigma_c^{5\times10^6}$ (MPa)	Fatigue limit
C55	$\sigma_f' = 694.44$ MPa $b = -0.079612$ $\varepsilon_f' = 0.33925$ $c = -0.51786$	250	73207	30630	242	192
St60-2*	$\sigma_f' = 592.34$ MPa $b = -0.05295$ $\varepsilon_f' = 0.3448$ $c = -0.52179$	380	2187	22191	315	197
St52-3**	$\sigma_f' = 619.13$ MPa $b = -0.070679$ $\varepsilon_f' = 0.3683$ $c = -0.6035$	340	2409	5283	330	178
AlCu4Mg	$\sigma_f' = 780.72$ MPa $b = -0.095672$ $\varepsilon_f' = 0.348$ $c = -0.861545$	312	695	–	100	167

* St60-2: 0.24 C; 0.44 Mn; 0.2 Si; 0.028 P; 0.012 S; 0.19 Cr; 0.06 Ni; Fe balance (wt.%)
** St52-3: 0.19 C; 1.4 Mn; 0.52 Si; 0.016 P; 0.015 S; 0.015 Al; Fe balance (wt.%)

ductility exponent), which are obtained by tests of cyclic loading of a material in the mode with a controlled strain. The area of the fatigue lifetime, when the elastic and plastic strain components in the overall strain are equally represented in fatigue lifetime (the so-called transitive number of cycles), corresponds to stresses lower than the static yield stress for most of the metals. If we take into consideration the fact that the fatigue process in a material is a result of a cyclic plastic strain, the approximation area of the stress--strain relation, described by the linear law (that is by non-admission of a plastic strain), corresponds during cyclic loading to much lower values of the stress than at the static stress point.

Looking for this limit at the cyclic stress point should be connected with a more sensitive measurement of the deformation processes which, by their irreversibility, cause material degradation and cumulation of fatigue damage.

The anelastic processes in materials obviously will not be significant in the areas of an order of magnitude of tens of MPa, but for a correct determination of the limit for using the relation (1) for the cyclic loading, the conventional material characteristics, such as the fatigue limit at harmonic stress, or the yield point at the static stress, cannot be utilized. The physical basis for its determination should be the limit of reversible and non-irreversible deformation processes in a cyclically loaded material.
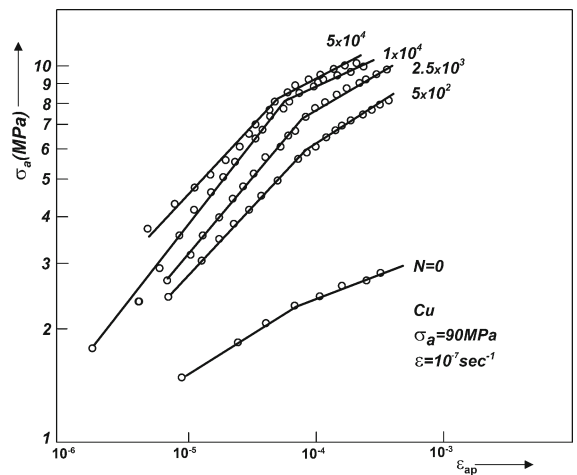

Fig. 1. Non-continuity on a cyclic stress-deformation curve [11, 12].

On closer investigation of the ratio σ_a-ε_{ap} (a cyclic stress-strain curve) described by the equation:

$$\sigma_a = k\,\varepsilon_{apl}^n, \qquad (2)$$

its non-continuity in the area of low amplitude values of the plastic strain was observed in Fig. 1.

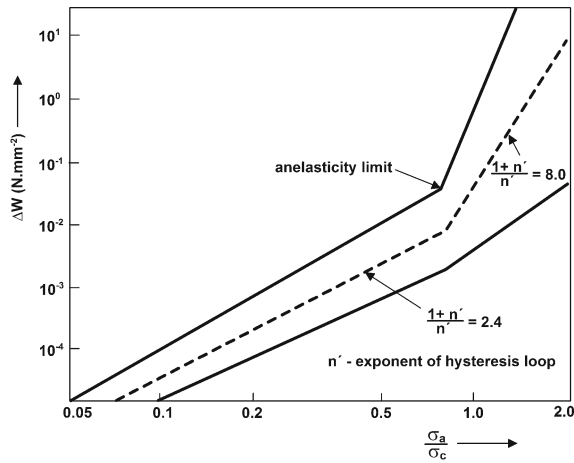This non-continuity can be expressed by a step change of the exponent n (Eq. (2)). While in the

Fig. 2. Non-continuity of the dependence of absorbed energy in one cycle ΔW on stress amplitude [11, 16].

Fig. 3. The hysteresis loop is unclosed at a harmonic amplitude of stress.

so-called "high cycle" area, its value does not exceed the value $n = 0.23$, and in the so-called "low cycle" area, the value $n = 0.18$ is below the breakpoint, the exponent value oscillates around $n = 0.5$ [12–15]. The same non-continuity was, as expected, confirmed also in the measurement of the energy absorbed in one loading cycle, depending on the stress amplitude (see Fig. 2).

Both identified non-continuities in the cyclic properties of the material are caused by the difference in the mechanism of plastic strain below or above the point of non-continuity. Based on the analysis of deformation processes, it is obvious that the amplitude of plastic strain ε_{ap} consists of the following components [11, 17, 18]:

– amplitude of the irreversible plastic strain ε_{api},
– amplitude of the reversible plastic strain ε_{apr}:

$$\varepsilon_{ap} = \varepsilon_{api} + \varepsilon_{apr}. \qquad (3)$$

The reversible plastic strain is achieved by a reversible movement of the dislocations where their arrangement is not changed within one loading cycle, and prevails precisely below the breakpoint. Increasing the loading amplitude of stress, the area of the hysteresis loop begins to grow and after completion of the loading cycle is closed. This indicates that the plastic strain is taking place by a reversible movement of dislocations. The occurrence of irreversible plastic processes can be observed by a sensitive measurement of the hysteresis loops (reading sensitivity of the deformation in the order of 10^{-7}). The permanent opening of the loop indicates the prevalence of irreversible plastic deformations. This phenomenon occurs in the area of stress amplitudes which corresponds well to the above mentioned non-continuity point on a cyclic stress-strain curve. Therefore, with an ad-
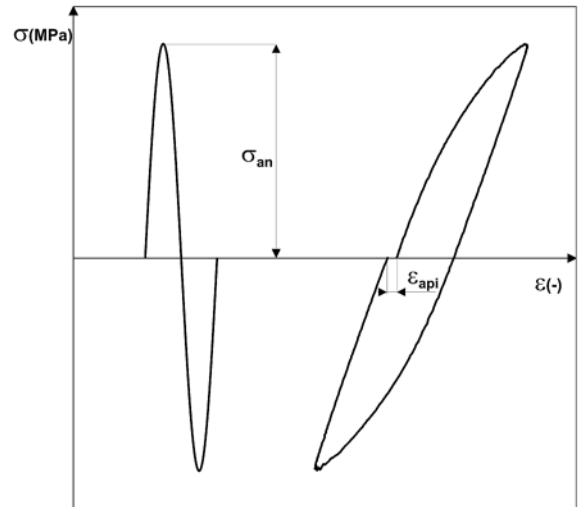
equate degree of approximation, the reversible component of plastic strain (prevailing below the break point of the cyclic stress-strain curve) can be considered as one which does not contribute to the damage, and the irreversible component (prevailing above the cyclic stress-strain curve) as contributing to the damage. The breaking point on the cyclic stress-strain curve (Fig. 1) as well as on the dependence of the absorbed energy per cycle (Fig. 2) is denoted as the cyclic anelasticity limit.

The cyclic anelasticity limit can be understood as a limit between reversible and irreversible deformation amplitudes, or a limit between the "damaging" and "non-damaging" deformation amplitudes. Macroscopically, the anelasticity point will be the smallest stress amplitude (σ_{an}), at which the material hysteresis loop will not close (value of irreversible plastic strain ε_{api} is non-zero, see Fig. 3).

3. Experimental materials and measurement procedures

The cyclic loading with sensitive measurement of the hysteresis loops (reading sensitivity of the deformation in the order of 10^{-7}) was performed on test specimens (see Fig. 4) of two cyclic stable metal materials – steel C55 and aluminium alloy AlCu4Mg.

The first material was steel C55 that is the silicon steel with the following chemical composition (see Table 2).

Another material was the aluminium alloy AlCu-4Mg that is the structural material commonly used in aircraft industry with the following chemical composition (see Table 3).

The specimens of these two materials were first cyclically loaded with the constant value of the stress

Table 2. The chemical composition of steel C55 (wt.%)

C	Mn	Si	P	S	Cr	Ni	Cu	Fe
0.54	0.64	0.25	0.016	0.015	0.17	0.04	0.06	mass residuum in %

Table 3. The chemical composition of the aluminium alloy AlCu4Mg (wt.%)

Cu	Mn	Si	Mg	Fe	Zn	Ti	Al
4.23	0.65	0.65	0.88	0.26	0.04	0.03	mass residuum in %

Fig. 4. Test specimens.

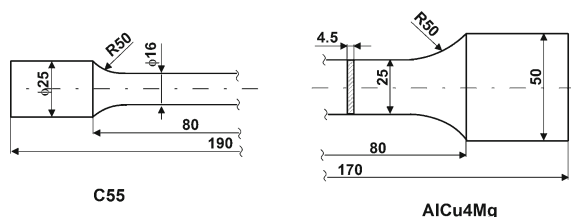Fig. 5. Examples of hysteresis loops measured at stress amplitudes 192–196 MPa, which represent the conventional fatigue limit of the steel C55.

amplitude that was 130 MPa repeatedly 10^5 number of cycles in order to span the phase of changing in the mechanical properties which could further distort the measurements. This was followed by loading of one cycle where the stress amplitude was changing gradually from the value of 135 to 215 MPa with the step of 10 MPa whereby every cycle was performed independently. During the measurement the force was registered simultaneously with the value of deformation and after each cycle the sensors were zeroed.

4. Results and discussion

Measurement of hysteresis loops in the area of the conventional fatigue limit which represented the stress amplitude for 5×10^6 cycles, in the relation

$$\sigma_{\mathrm{a}} = \sigma_{\mathrm{f}}'(2N_{\mathrm{f}})^b \qquad (4)$$

showed that at these amplitudes of load, the loops are exclusively unclosed ones (Fig. 5). This confirms the fact that at the levels of loading in the area of stresses at the conventional fatigue limit, the irreversible deformation processes are fully developed. These results can also explain the phenomenon often seen in practice of a considerable variance in fatigue test results in obtaining the parameters of Eq. (4) in the area of the lifetime on the order of 10^6 cycles until fracture. Some tests are unfinished (no fatigue fracture occurred), and at the same time other tests are completed (there was a fatigue fracture). The number of unclosed loops also shows a variance (Fig. 5), with
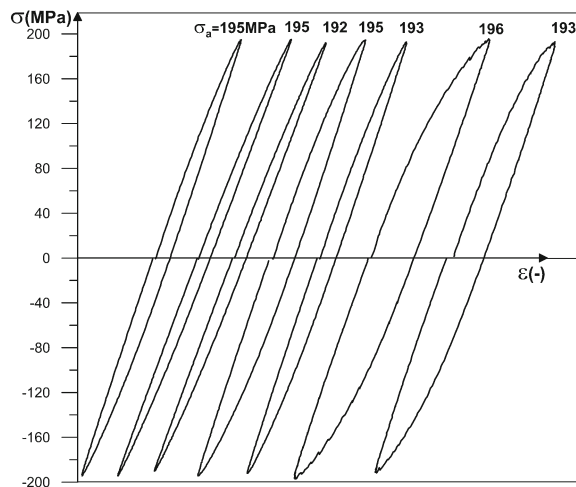
the consequence that the test specimen not ended by fatigue fracture belonged to those with a smaller loop opening. The fatigue fracture could in such cases occur at a higher number of loading cycles than it occurred in the test (or in the case of continued testing).

The real beginning of irreversible deformation processes can be seen in Fig. 6. For the steel C55, a measurable opening of hysteresis loops can be observed at stress amplitudes from 143 to 170 MPa. Such stress amplitudes represent 79 to 92 % of the fatigue limit for 3×10^6 cycles until fatigue fracture. These partial results are in compliance with the measurements of other authors [11, 16].

Preliminary results indicate that the cyclic anelasticity limit which could be a physically justified limit for using a linear dependence (Eq. (1)) for stress-strain relation at cyclic loading, is located significantly below the conventional material fatigue limit. It documents also the state of specimen surface of material C55 after cyclic loading (Fig. 7): cross-section 1 loaded with the amplitude of 282 MPa and cross-section 2 loaded with amplitude of 188 MPa as well as the cross-section 3
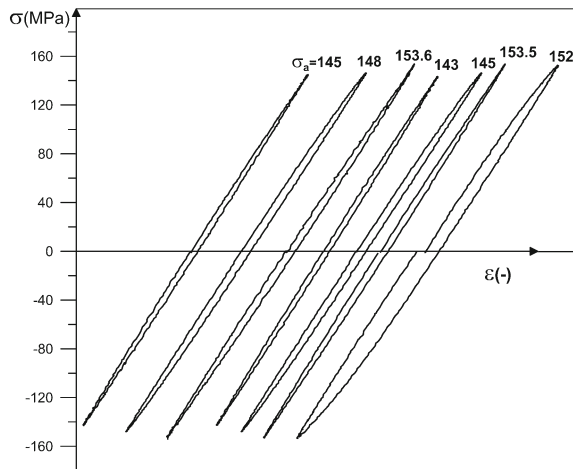
Fig. 6. Examples of hysteresis loops measured at stress amplitudes 143–153.6 MPa, which represent the beginning of irreversible deformation processes of the steel C55.

loaded with the amplitude of 132 MPa.

Another question mark, when using Hooke's law for cyclic loading, is the constancy of Young's modulus of elasticity E. When loaded cyclically, the constant of proportionality E is changed depending on the kind of material tested (cyclically softening/strengthening itself, or cyclically stable).

During loading by a range of random amplitude of force, the modulus of elasticity of the material was measured and evaluated, both in tensile and pressure loop parts, for stress values up to 100 MPa. The results for the C55 material are in Table 4.

When comparing the sets of measured values of the modulus of elasticity obtained in 15 % of the total time of the fatigue lifetime, and 40 % of the total time of the fatigue lifetime, the maximum change in Young's modulus E was 3.5 %. By comparing these sets using both the F-test and the t-test, applying 95 and 99 % significance level, this change was statistically negli-
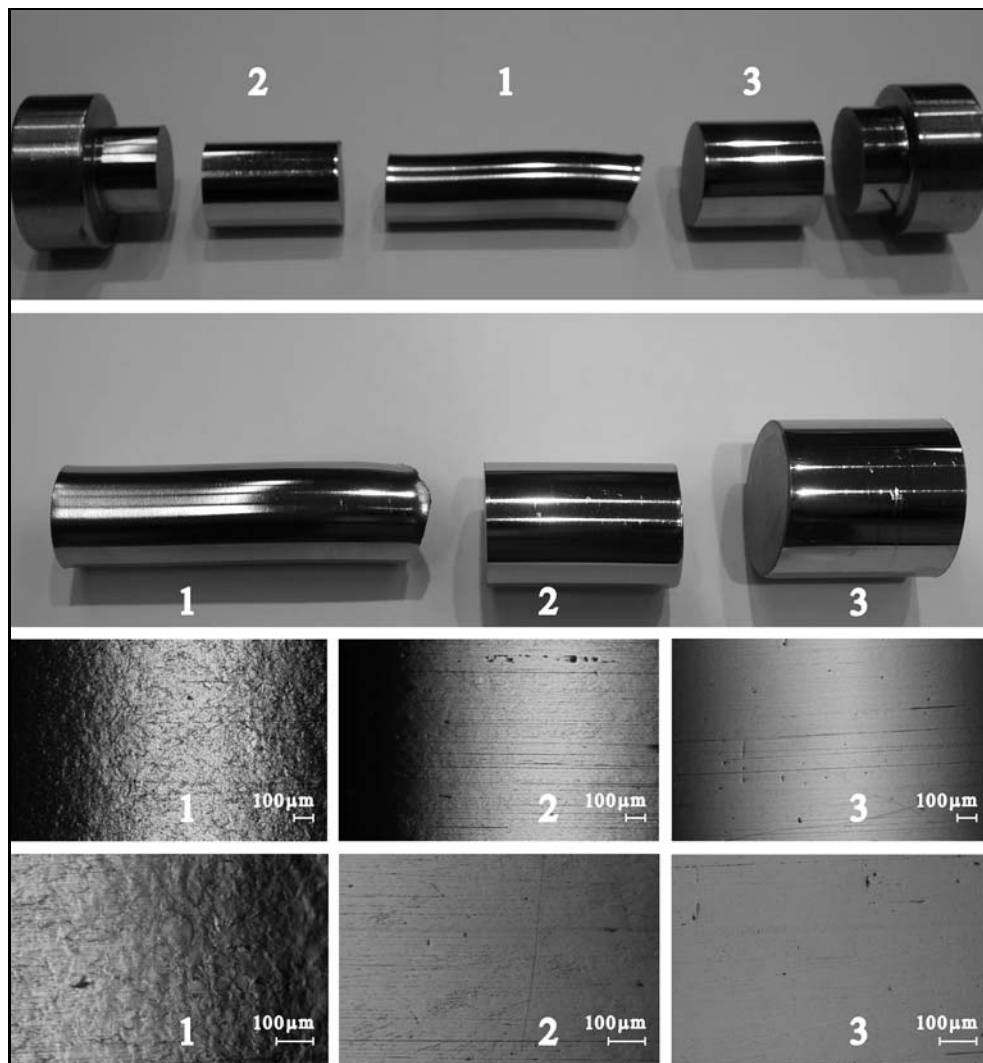

Fig. 7. 1 – markedly deformed surface (loading with the stress amplitude of 282 MPa), 2 – surface with the marks of cyclic deformation (loading with the stress amplitude of 188 MPa – below the conventional fatigue limit), 3 – surface without any marks of cyclic deformation (loading with the stress amplitude of 132 MPa – below the cyclic anelasticity limit).

T a b l e 4. Young's modulus during the cyclic loading steel C55

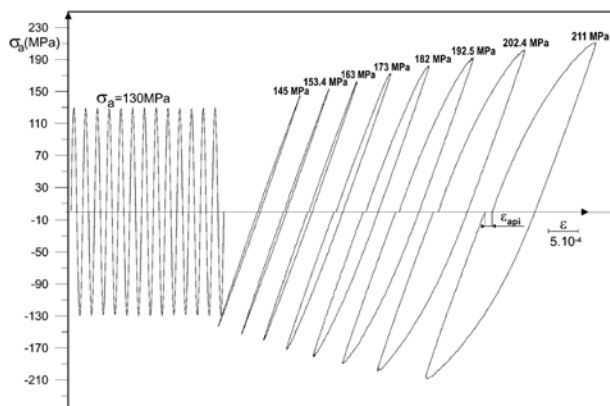|	Tensile part	Compressive part
Range of variance the Young's modulus at set of specimens	$(1.96–2.6) \times 10^5$ MPa	$(1.92–2.6) \times 10^5$ MPa
Average of Young's modulus at 15 % of N_f	220166 MPa	216094 MPa
Average of Young's modulus at 40 % of N_f	213697 MPa	209229 MPa
Change of Young's modulus in percentage during loading single specimen	2–6.5 %	2–8 %

Fig. 8. A hysteresis loop development at the beginning of irreversible plastic deformations (measured on a C55 steel specimen after loading 10^5 cycles with stress amplitude 130 MPa).

gible, i.e. these sets can be considered as identical – the steel was cyclically stable.

Similarly, the sets of measured Young's moduli E in tensile and compressive parts of hysteresis loops were compared. After application of the F-test for the difference or compliance of the two sets of variations, and the application of the t-test for the difference or compliance of the mean values of the two sets, their difference was statistically insignificant for 95 % level of significance.

5. Conclusions

Based on the results presented, it is possible to draw these partial conclusions:

– In the investigated material, the gradient of the stress-strain of the hysteresis loop for random sequence of amplitudes is the same for tensile and compression stress and equals Young's modulus of material elasticity. These conclusions are in good compliance with reality for materials cyclically stable and with a short period of change in their mechanical properties.

– The part of the stress-strain curve at cyclic loading can be approximated by Hooke's linear law, with the proportionality constant E.

– The limit of Hooke's law utilization at cyclic loading – denoted as the anelasticity limit (Fig. 8) – is

situated below the conventional fatigue limit.

Changes in Young's modulus of elasticity can, however, be seen in the measured results of every fatigue curve of the material, and so this change at cyclic loading occurs in the final variation of the computed estimation of the fatigue lifetime [19].

References

[1] Zener, C.: Elasticity and Anelasticity of Metals. Chicago, Illinois, The University of Chicago Press 1960.
[2] Barrett, Ch. S.: Structure of Metals. New York, McGraw-Hill 1952.
[3] Sapozhnikov, K., Golyandin, S., Kustov, S.: Materials Science and Engineering A, *521–522*, 2009, p. 109. doi:10.1016/j.msea.2008.10.077
[4] Blanter, S., Golovin, I. S., Neuhäuser, H., Sinning, H.-R.: Internal Friction in Metallic Materials. Berlin, Heidelberg, Springer-Verlag 2007.
[5] Yagmur, L., Fank, S., Aydemir, B.: Sensors and Actuators A, *136*, 2007, p. 261. doi:10.1016/j.sna.2006.10.030
[6] Golovina, I. S., Neuhäuserb, H., Rivièrec, A., Strahl, A.: Intermetallics, *12*, 2004, p. 125. doi:10.1016/j.intermet.2003.10.003
[7] Gremaud, G., Kustov, S.: Physical Review B, *60*, 1999, p. 9353. doi:10.1103/PhysRevB.60.9353
[8] Mishra, R. S., Mukherjee, A. K.: In.: Proceedings of Symposium Creep Behaviour of Advanced Materials for the 21st Century. Eds.: Mishra, R. S., Mukherjee, A. K., Murty, K. L. Materials Park, The Minerals, Metals & Materials Society 1999, p. 503.
[9] Bílý, M.: Cyclic Deformation and Fatigue of Metals. Oxford, Elsevier Science Ltd. 1993.
[10] Yagmur, L., Gülmez, T., Hacızade, F., Kaçmaz, S.: Review of Scientific Instrumentation, *76*, 2005, p. 093903. doi:10.1063/1.2038127
[11] Klesnil, M., Lukáš, P.: Fatigue of Metallic Materials. Oxford, Elsevier Science Ltd. 1992.
[12] Langraf, R. W.: ASTM Special Technical Publication, *467*, 1970, p. 3.
[13] Feltner, C. E., Morrow, J. D.: Trans. ASME Series D, *83*, 1961, p. 15. doi:10.1115/1.3658884
[14] Lukáš, P., Klesnil, M., Polák, J.: Materials Science and Engineering, *15*, 1974, p. 239. doi:10.1016/0025-5416(74)90057-3
[15] Lukáš, P., Klesnil, M.: Phys. Stat. Sol., *21*, 1967, p. 717. doi:10.1002/pssb.19670210233
[16] Lazan, B. J.: Damping and Resonance Fatigue Properties of Materials Considering Elevated Temperatures. New York, McGraw-Hill 1961.

[17] Chmelko, V.: In: Proceedings of the 6th International Conference "Akademická Dubnica 2000". Trnava, STU 2000, p. 51. ISBN 8022714488 (in Slovak).

[18] Chmelko, V.: In: Proceedings of the 3rd International Conference "Applied Mechanics 2001". Pilsen, The University of West Bohemia 2001, p. 103. ISBN 8070827351 (in Slovak).

[19] Kliman, V.: Kovove Mater., *37*, 1999, p. 18.