

# Calculation of the phase content of a steel plate at local heating

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## Abstract

The model is proposed to describe the phase content formation in the low alloy steels subjected to monotonous cooling. In particular, a procedure is developed to investigate the phase content of thin steel plates heated by distributed moving heat sources with several centres of localization. It is shown that the proper choice of the intensity and location of the centres of heating makes it possible to halve the maximum content of martensite and substantially decrease the non-uniformity of its space distribution in the zone of thermal influence.

**Key words:** martensite and bainite formations, heat conductivity, mathematical modelling, moving heat sources

## 1. Introduction

Various kinds of steel are widely used in engineering because they provide the required level of reliability while keeping other important properties. During a heating-cooling process, the solid steels undergo phase transformations. Properties of a material and consequently the properties of a final product depend on its chemical content and structural state. At given chemical content of a material, the phase state of a solid as well as its time evolution determines functional properties of the product [1–3]. Low-alloy steels subjected to local heating undergo phase transformations in limited volumes. It invokes spatial non-homogeneity of such important material properties as reliability, hardness, resistance to corrosion and leads to arising of residual stresses.

Therefore, there is a need in developing optimum thermal processing techniques using known heating sources in order to obtain a product of the required properties. Developing of such techniques should be grounded on suitable mathematical models to capture coupled physical and mechanical processes accompanying the process of the global or local thermal processing. Such models would facilitate comprehens-

ive analysis of all the important processes in a steel product dependent on the characteristics of a heat source and other heating conditions. Several approaches to solving this problem are reported in [4–6]. In this study, we propose a mathematical model to describe evolution of the formation of the structural state of steel products subject to gradual cooling. The developed model can be used for tests of the effect of normally spatial distributed moving heat sources on the structural state of a steel plate. Such heat sources model standard industrial devices like gaseous, inductive and electric heaters [7]. In such a case, it is important to investigate and optimize phase content and stress state at mechanical and thermal loadings due to existing heating regimes. The problem is related to the development of local heating technologies. They could be used as substitute of general heating as well as in specific cases. In welding technologies, the heating by distributed heat sources is widely used. Here we apply this idea in order to influence the process of heat transfer in the body and, as a consequence, the phase content formation in local domains of the body. This influence can be achieved by the control of heat sources power, their spatial localization and moving velocity. In practice, so-called circular normally dis-

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tributed moving heat sources are used for this purpose. They are characterized by the normal (Gauss) distribution of heat flux power. In particular, such distributions are provided by ordinary gaseous, inductive or electric heaters [6]. Then the goal of optimization could be calculation of both the phase content of a product itself and the parameters determined by the phase content, for example the value and distribution of residual stresses.

The general approach to solving the mentioned problems consists in involving kinetic equations describing polymorphic transformations in solids [8, 9]. However, there are no standard methods to get these equations.

In many technological processes the temperature in each point of the solid changes gradually during formation of non-equilibrium phase content. For these cases the statistical data are correlated with the parameters of the phase content of the low alloy steels on cooling [10]. In this study, the well-known mathematical model to capture formation of the spatially non-homogeneous phase content of steels is used for analysis and optimization of the phase content. The model involves using statistical data. The model allows to calculate parameters of the phase content and the associated residual stresses in steel products subject to certain heat sources.

## 2. Phase content formation model

Equations of the model are obtained in three stages. At the first stage, the thermal conductivity equation is written along with boundary heat exchange conditions corresponding to the technological process under investigation. Solutions of the thermal conductivity equation being found, duration of the material staying in the temperature range responsible for phase content formation is calculated. At the second stage, statistical approach is involved to the description of the material phase content dependent on its chemical state and duration of material staying in above mentioned temperature range. In doing so, at given cooling conditions and calculated duration of material staying within temperature range 850–500 °C, the content of the martensite, bainite and ferrite-perlite phases in each point of the steel body is determined. At the last stage, when phase content of the steel body is known, the continuum mechanics equations are involved to determine stressed state of the body.

The key parameter used in the mentioned approach is a time interval when material temperature falls within the range corresponding to complete austenization and minimum stability of austenite. Because of that, supposing known solutions of the heat conduction equation, the duration  $\tau^*$  of cooling from 850 °C

to 500 °C for points satisfying condition  $T' > A_{c3}$  and duration  $\tau^{**}$  of cooling from  $T'$  to  $T_m$  for points satisfying condition  $A_{c1} < T' < A_{c3}$  ( $A_{c1}$  is the temperature of eutectic transformation of the steel) are calculated using the following equations:

$$T(x, \tau_1) = 850 \text{ °C}, T(x, \tau_2) = 500 \text{ °C}, \tau^* = \tau_2 - \tau_1 \quad (1)$$

for each point of the solid staying at the initial (before cooling) temperature  $T' > A_{c3}$ , and

$$T(x, \tau'_1) = T', T(x, \tau_2) = 500 \text{ °C}, \tau^{**} = \tau_2 - \tau'_1 \quad (2)$$

for each point of the solid falling in the temperature range  $A_{c1} < T' < A_{c3}$ . Here  $\tau_1, \tau_2, \tau'_1$  are moments of time when the temperatures  $T = T'$  are achieved, respectively.

At the second stage the times  $\tau^*$  and  $\tau^{**}$  found for each point are involved in calculation of contents of martensite M, bainite B and ferrite-perlite FP in this point. For this purpose the following correlations are used:

$$M(\tau^0) = 100\beta(1 - \Phi(\ln(\tau^0/\tau_M)/\ln S_M)),$$

$$(FP)(\tau^0) = 100\beta(1 - \Phi(\ln(\tau^0/\tau_{FP})/\ln S_{FP})) + 100(1 - \beta), \quad (3)$$

$$B(\tau^0) = 100 - M(\tau^0) - (FP)(\tau^0),$$

where  $M(\tau^0)$ ,  $(FP)(\tau^0)$ ,  $B(\tau^0)$  denote respectively martensite, ferrite-perlite and bainite content in the point of interest;  $\Phi(\tau^0)$  is the structure component of the normal distribution function dependence on the general cooling duration  $\tau^0$  in the temperature range  $A_{c3} - T_m$ ;  $\beta$  is the coefficient accounting for equilibrium phase content at cooling down from the temperature  $A_{c1} < T' < A_{c3}$ ;  $\tau_M, \tau_{FP}, S_M, S_{FP}$  are the coefficients determined by the polynomial functions of the steel phase content. The latter are given by formulas [10]:

$$\ln \tau_M = -2.10 + 15.50C + 0.96Mn + 0.84Si + 0.77Cr + 0.74Mo + 0.7Ni + 0.3V + 4.0Al + 0.5W + 0.8Cu - 13.5C^2,$$

$$\ln S_M = -0.56 - 0.41C + 0.10Mn + 0.14Cr + 0.30Mo + 2.70Ti + 1.1Nb + 0.5Cu - 1.7CMo, \quad (4)$$

$$\ln \tau_{FP} = 0.34 + 5.20C + 1.8Mn + 0.53Si + 0.33Cr + 2.90Mo + 1.30Ni + 1.50W + 1.0Cu - 5.10C^2,$$

$$\ln S_{FP} = 0.91 - 0.90C + 0.09Mn + 0.08Cr + 0.34Mo + 0.15Ni + 0.85V + 2.2Ti + 0.43W.$$

In [10], the above correlations are obtained at  $\beta = 1$  for the conditions of homogeneous cooling. In our model it is supposed that correlations (3) and (4)

are valid for the practical cases of arbitrary spatial temperature distributions. Such an assumption could be treated in a way similar to the statement about local thermodynamic equilibrium. The cooling may start from any temperature including one corresponding to complete austenite state ( $T' > A_{c3}$ ). Martensite and bainite states would appear if cooling starts from state with some initial austenite content. In the model, we assume that while cooling from the initial state is characterized by austenite content equal to  $\beta$ , martensite and bainite contents are proportional to initial austenite content while ferrite-perlite content consists of two parts, namely the part at the start of cooling  $100(1 - \beta)$  and the part appearing at austenite decay.

The value  $\beta$  of austenite initial content can be calculated from a steel state diagram. In our model we adopt approximation for the  $\beta$  dependence on the initial cooling temperature  $T'$  for the range of polymorphic transformations corresponding to temperature range  $A_{c1} \leq T' \leq A_{c3}$ :

$$\beta = \begin{cases} 1, & \text{for } T' > 850^\circ\text{C}, \\ \frac{(850^\circ\text{C} - T')}{(850^\circ\text{C} - A_{c1})}, & \text{for } A_{c1} \leq T' \leq 850^\circ\text{C}, \\ 0, & \text{for } T' < A_{c1}. \end{cases} \quad (5)$$

Phase content of a material in certain point is determined by the cooling duration in the temperature range  $A_{c3} - T_m$ . This phase content can be associated with the averaged cooling rate in that temperature range. If the cooling starts from temperature exceeding  $A_{c3}$  then averaged cooling rate can be written as  $\nu_c = \frac{A_{c3} - T_m}{\tau^*}$ . If cooling starts from the temperature  $T'$  ( $A_{c1} \leq T' \leq A_{c3}$ ) when initial austenite content falls in the range (0; 1), this rate is

$$\nu'_c = \frac{T' - T_m}{\tau^{**}}, \quad (6)$$

where  $\tau^{**}$  is the cooling duration from  $T'$  to  $T_m$ .

For the points cooled from the temperature  $T'$  ( $A_{c1} \leq T' \leq A_{c3}$ ) we introduce conditional cooling duration  $\tau_c^*$  equal to theoretical cooling with the rate  $\nu'_c$  in the range  $A_{c3} - T_m$ . Then

$$\tau_c^* = \tau^{**} \frac{A_{c3} - T_m}{T' - T_m}. \quad (7)$$

For cooling from arbitrary temperature  $T'$  we assume

$$\tau^* = \begin{cases} \tau_c^*, & \text{for } T' > A_{c3}, \\ \tau_c^*, & \text{for } A_{c1} < T' < A_{c3}. \end{cases} \quad (8)$$

The points do not undergo phase transformations if they are cooled from the initial temperature less than  $A_{c1}$  ( $M = 0\%$ ,  $B = 0\%$  is taken for these points).

At the third stage, we assume analogy between influences of the thermal and phase dilatation on the body's volume. Therefore, we calculate stresses due to cooling by the expression:

$$\sigma_{ij} = \frac{E}{1 + \nu} \left[ e_{ij} + \frac{1}{(1 - 2\nu)} \left( \nu e - (1 + \nu) \frac{e_a}{3} \right) \delta_{ij} \right], \quad (9)$$

where  $E$  is the Young's modulus,  $\nu$  is the Poisson ratio,  $\delta_{ij}$  is the Kronecker delta,  $\sigma_{ij}$ ,  $e_{ij}$  are the components of the stress and strain tensors,  $e = e_{11} + e_{22} + e_{33}$ ,  $e_a$  is the general dilatation (caused by changes in interatomic distances and in the phase content of a steel). For complete cooling of steel products we have

$$e_a = \beta_M \xi_M + \beta_B \xi_B, \quad (10)$$

where  $\xi_M = \frac{M}{100}$ ,  $\xi_B = \frac{B}{100}$ ;  $\beta_M$ ,  $\beta_B$  are the bulk dilatation coefficients. Completing Eqs. (4)–(10) by equilibrium equations we obtain the set of models equations to capture phase content and stress state in a steel product.

### 3. Mathematical problem

Let us consider a thin plate, 10 mm in thickness. The plate is heated by a set of moving normally circular heat sources (sources satisfying the Gauss law of distribution for heat flux power) having three localization centres (Fig. 1). In the figure the index "O" denotes principal localization centre, the index "x" denotes additional localization centres; the arrow indicates the moving direction. The more powerful principal heat source heats the plate until given technological heating temperature is achieved. The less powerful additional heat sources slow down the cooling, assuring decreased martensite content on cooling.

We propose the method of thermal heating of a plate optimized by certain criterion. The constraints

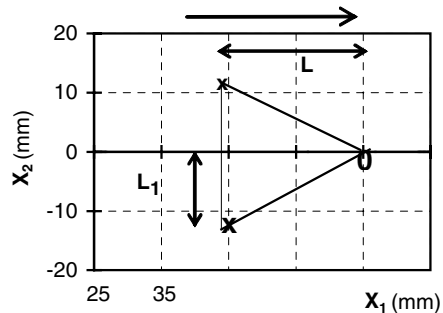


Fig. 1. Schematics of the moving heat source having three localization centres; arrow ( $\rightarrow$ ) – the moving direction of the system.

on the method usage are found (for above model of heating sources).

The temperature distribution in the plate is determined by 2D heat conduction equation for thin plates [11]:

$$c\rho \left( \frac{\partial T}{\partial \tau} \right) = \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right) - \alpha^*(T - t_c) + \bar{Q},$$

$$(j = \bar{1}, \bar{2}) \quad (11)$$

at the following boundary

$$\begin{aligned} \partial T / \partial x_1 &= 0 \quad \text{for } x_1 \rightarrow \pm\infty, \\ \partial T / \partial x_2 &= 0 \quad \text{for } x_2 \rightarrow \pm\infty \end{aligned} \quad (12)$$

and initial conditions

$$T(x_1, x_2, \tau) = t_0 \quad \text{for } \tau = 0. \quad (13)$$

Here  $T = 1/2h \int_{-h/2}^{h/2} t dx_3$  is the temperature averaged over plate thickness,  $\tau$  is the time,  $x_1, x_2$  are coordinates of the middle plate surface,  $x_3$  is the coordinate normal to the plate surface,  $\lambda$  is the thermal conductivity coefficient,  $c$  is the specific heat,  $\rho$  is the density,  $t_c$  is the ambient temperature,  $t_0$  is the initial plate temperature,  $\alpha^* = 2\alpha/h$ , where  $\alpha$  is the heat exchange coefficient,  $\bar{Q}$  is the heat source power per unit thickness,  $\bar{Q} = Q_0 + Q$ , where  $Q_0$  is the source of hidden power,  $Q$  is the heat source power.

The heat source  $\bar{Q}$  consists of two parts of different nature.  $Q_0$  counts for heat absorption (generation) during phase transformations.  $Q$  is the external heat source the parameters of which (location, power, etc.) are determined by the heating objective.  $Q_0$  and  $Q$  are time and coordinates dependent. The source  $Q$  is capable to heat the body up to temperature 200–1000 °C. Heat sources due to polymorphic transformations are known to be able to cause the temperature change up to 30 °C [12]. Therefore, in modelling real technologies of phase content formation the condition  $\bar{Q} \sim Q$  can be accepted.

The temperature dependence of the heat exchange coefficient  $\alpha$  is taken into account, which allows to consider all possible heat exchange conditions, including thermal radiation exchange. Hidden power of phase transition is counted for by introducing additional heat source.

The heat produced by moving heat sources is determined by the following expressions:

$$\begin{aligned} Q &= (Q_1^*/2\pi h\sigma) \exp\left(\frac{(x_2^2 - (x_1 - x_1^*)^2)}{2\sigma}\right) + \\ &+ (Q_2^*/2\pi h\sigma) \exp\left(\frac{((x_2 - x_2^*)^2 - (x_1 - x_1^*)^2)}{2\sigma}\right) + \\ &+ (Q_2^*/2\pi h\sigma) \exp\left(\frac{((x_2 + x_2^*)^2 - (x_1 - x_1^*)^2)}{2\sigma}\right), \end{aligned} \quad (14)$$

where  $x_1 = v\tau$ ,  $x_1^* = v\tau - L$ ,  $x_2^* = x_2 + L_1/2$ ,  $\sigma$  is the scattering parameter,  $v$  is the speed of moving sources. Heat flow from each heat source is assumed to have Gauss distribution.  $Q_1^*$  and  $Q_2^*$  are the powers of principal and additional heat sources.

The problem of phase content calculation was solved by the finite element method using weighted residuals method.

Using solutions of the heat conduction equation and steel phase content kinetics equations, the phase content spatial distribution in the plate subject to moving heat sources is calculated. The position and power of heat sources are found assuring minimal martensite content in the plate.

#### 4. Numerical results

Parametric optimization, that is, the selection of heat sources parameters (powers of heat sources  $Q_1^*$  and  $Q_2^*$ , their locations) and heating conditions (heat exchange coefficient, ambient temperature), is used for the choice of local heating regime optimum in terms of the phase content. Optimization of the phase content is carried out by a criterion of minimum of maximal martensite phase content in the area of local heating.

Temperature field isotherms and phase content distributions in the plate at  $Q_1^* = 800 \text{ kW m}^{-1}$ ,  $v = 0.002 \text{ m s}^{-1}$  and the values of geometric parameters  $L_1 = 0.005 \text{ m}$  and  $L = 0.020\text{--}0.025 \text{ m}$ ,  $\sigma = 3 \times 10^{-5} \text{ m}^2$  are shown in Figs. 2, 3. Isotherms 1, 2 and 3 correspond to temperatures 500 °C, 723 °C and 850 °C, respectively, while numbers 4, 5 and 6 correspond to martensite, bainite and ferrito-perlite phase contents. Conditions of gradual cooling ( $\frac{\partial T}{\partial \tau} < 0$ ) for each point of the plate are observed. One can check by the isotherms shape if these conditions are observed – at certain distance from the line of the principal heat source movement the points exist which meet the same isotherm twice during cooling (Fig. 4).

If additional heat sources are excessively powerful the cooling of the plate becomes non-gradual thus making the above approach not applicable. In this case the cooling takes more time due to temperature rise after initial short time drop of the temperature of the plate. Under these conditions the isotherms are characterized by variable curvature (Fig. 4).

Table 1 presents the calculated values of the maximum martensite phase content in the heating area dependent on geometric parameters  $L$  and  $L_1$  as well as the power of additional sources  $Q_2^*$ . The value  $L = 0.025 \text{ m}$  was found to assure optimum martensite content. Table 2 contains the maximum martensite and bainite contents in the steel at the constant value  $L = 0.025 \text{ m}$  and variable values  $L_1$  and  $Q_2^*$ .

It was found that the minimum martensite phase

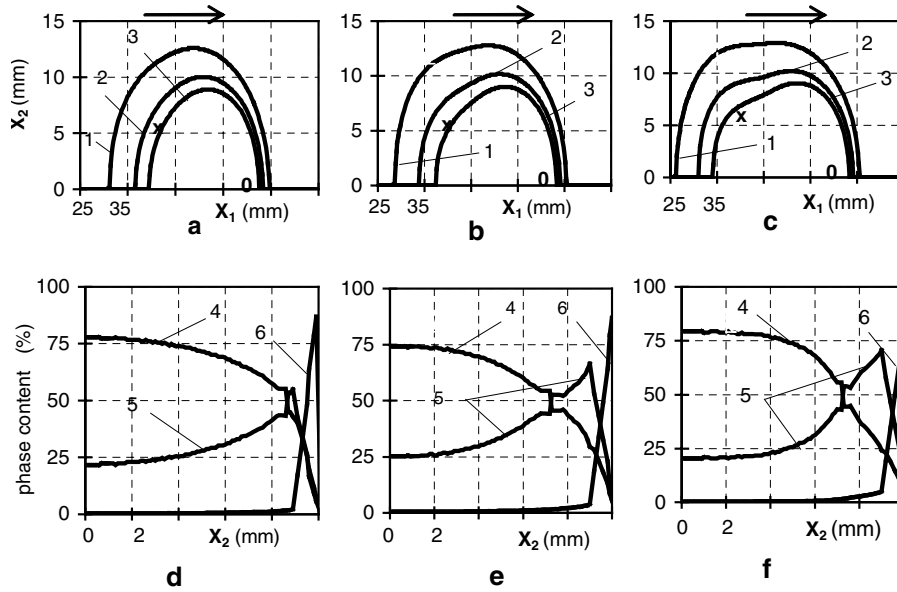


Fig. 2. The isotherms (1, 2, 3) and phase distributions (4, 5, 6) in the plate at  $L = 0.020$  m and  $L_1 = 0.005$  m and different power  $Q_2^*$ : (a, d)  $Q_2^* = 40$  kW m<sup>-1</sup>; (b, e)  $Q_2^* = 80$  kW m<sup>-1</sup>; (c, f)  $Q_2^* = 120$  kW m<sup>-1</sup>; arrow (→) – the moving direction of the system.

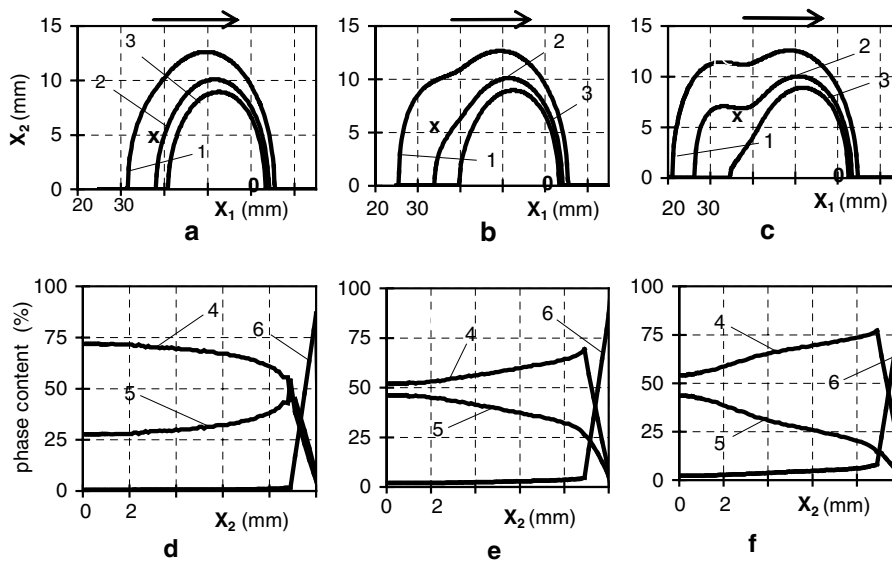


Fig. 3. The isotherms (1, 2, 3) and phase distributions (4, 5, 6) in the plate at  $L = 0.025$  m and  $L_1 = 0.005$  m and different power  $Q_2^*$ : (a, d)  $Q_2^* = 40$  kW m<sup>-1</sup>; (b, e)  $Q_2^* = 100$  kW m<sup>-1</sup>; (c, f)  $Q_2^* = 160$  kW m<sup>-1</sup>; arrow (→) – the moving direction of the system.

Table 1. The maximum martensite and bainite contents at  $L_1 = 0.005$  m,  $v = 0.002$  m s<sup>-1</sup>,  $Q_1^* = 800$  kW m<sup>-1</sup> and different values of  $Q_2^*$  and  $L$

$Q_2^*$ (kW m <sup>-1</sup> )	$L = 0.020$ m		$L = 0.025$ m		$L = 0.030$ m	
	$M_{\max}$	$B_{\max}$	$M_{\max}$	$B_{\max}$	$M_{\max}$	$B_{\max}$
40	77.9	21.7	71.9	27.6	81.1	18.7
60	74.9	24.7	61.4	37.3	75.5	24.1
80	74.2	25.3	50.0	48.5	71.6	27.9
100	76.7	22.9	46.0	52.0	–	–
120	79.4	20.3	41.7	56.0	–	–
140	81.6	18.1	39.0	58.4	–	–
160	83.4	16.4	43.6	54.2	–	–

Table 2. The maximum martensite and bainite contents at  $L = 0.025$  m,  $v = 0.002$  m s<sup>-1</sup>,  $Q_1^* = 800$  kW m<sup>-1</sup> and different values of  $L_1$  and  $Q_2^*$

$Q_2^* \setminus L_1$		0.005 m	0.006 m	0.007 m	0.008 m	0.009 m
40 kW m <sup>-1</sup>	$M_{\max}$	71.9	73.2	73.9	76.1	78.2
	$B_{\max}$	27.6	26.3	25.6	23.5	21.4
60 kW m <sup>-1</sup>	$M_{\max}$	61.4	62.5	64.8	68.1	71.9
	$B_{\max}$	37.3	36.6	34.4	31.3	27.6
80 kW m <sup>-1</sup>	$M_{\max}$	50.0	50.0	55.2	59.1	65.2
	$B_{\max}$	48.5	48.6	43.5	39.9	34.1
100 kW m <sup>-1</sup>	$M_{\max}$	46.0	50.0	50.0	50.0	57.1
	$B_{\max}$	52.0	48.1	48.3	48.5	41.7
120 kW m <sup>-1</sup>	$M_{\max}$	41.7	41.7	43.3	45.2	50.0
	$B_{\max}$	56.0	56.0	54.6	52.8	48.4
140 kW m <sup>-1</sup>	$M_{\max}$	39.0	38.6	38.6	40.9	45.6
	$B_{\max}$	58.4	58.7	58.7	56.7	52.4
160 kW m <sup>-1</sup>	$M_{\max}$	43.6	37.1	<b>35.6</b>	36.7	40.1
	$B_{\max}$	54.2	60.1	<b>61.4</b>	60.4	57.4

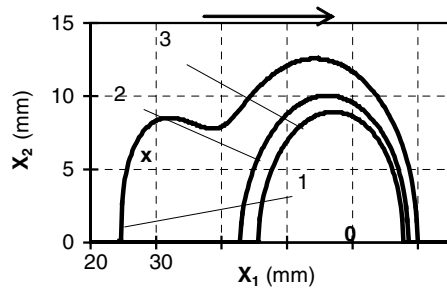


Fig. 4. The isotherms in the plate at  $Q_2^* = 120$  kW m<sup>-1</sup>;  $L = 0.030$  m and  $L_1 = 0.005$  m when the proposed model is not applied for calculation of the phase content; arrow (→) – the moving direction of the system.

content ( $M_{\max} = 35.6$  %,  $B_{\max} = 61.4$  %) in the area of local heating is obtained for the following values of the power and geometric parameter:  $Q_1^* = 800$  kW m<sup>-1</sup>,  $Q_2^* = 160$  kW m<sup>-1</sup>,  $L = 0.025$  m,  $L_1 = 0.007$  m (Table 2).

The calculated values of the phase content might be useful for determination of residual structural stresses. The case of general cooling of a steel bar of squared cross section was studied in [13].

## 5. Conclusions

Calculations have been carried out using numerical parameters, values of which are close to those currently used at existing underwater welding technologies [14]. It was shown that the phase content of the

heated area of steel can be considerably influenced by the selection of geometric parameters and the power of moving circular heat sources. The martensite phase content can be reduced twice compared to the case of using heating set not involving additional sources (in Tables 1, 2). In doing so, the temperature in the centre of heating area remains the same. The gradient of non-equilibrium phase distributions in the heating area can also be considerably reduced thus assuring decreased stresses concentration.

The proposed method can be applied for development optimum, in terms of the phase content as well as other parameters, global and local heating-cooling of low alloy steel products and for calculation of structural stresses.

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