CYCLIC STRESS-STRAIN CURVE UNDER RANDOM LOADING

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The authors define the random cyclic stress-strain curve as a set of centred closed hysteresis loop peaks evaluated by the Rain Flow Method from the stress-strain histories of randomly loaded cyclically softening low carbon steel specimens. They compare the numerical values of the cyclic stress-strain curve parameters obtained under constant amplitude sinusoidal loading, macroblock loading, and random loading and find that whereas the first two methods yield practically identical values, random loading produces higher values. This fact is discussed.

CYKLICKÁ DEFORMAČNÁ KRIVKA PRI NÁHODNOM NAMÁHANÍ

V príspevku definujeme cyklickú deformačnú krivku pri náhodnom namáhaní ako množinu vrcholov centrovaných uzavretých hysteréznych slučiek vyhodnotených metódou tečúceho dažďa z náhodného procesu napätie-deformácia cyklicky sa zmäkčujúcej nízkouhlíkovej ocele. Porovnávame číselné hodnoty parametrov cyklických deformačných kriviek získaných pri harmonickom, blokovom a náhodnom zaťažovaní a konštatujeme, že zatiaľ čo prvé dve krivky sú prakticky identické, parametre cyklickej deformačnej krivky pri náhodnom zaťažovaní sú vyššie. O tomto výsledku diskutujeme.

1. Introduction

There is no doubt that the knowledge of behaviour of a material subjected to cyclic loading is crucial for its fatigue life assessment. As usually, it is described by experimentally derived and verified correlation between the cyclic strength coefficient k and the cyclic strain hardening exponent n, called the cyclic stress-strain curve (CSSC) in the form

$$\sigma_{\mathrm{a}} = k\varepsilon_{\mathrm{ap}}^{n}, \tag{1}$$

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where σ_a is the stress amplitude and ε_{ap} is the plastic strain amplitude of sinusoidal loading.

A number of the CSSCs for various materials have been published (see, e.g., [1]). They have been most often evaluated from the tests under constant sinusoidal strain amplitude or the constant plastic strain amplitude. It is common to call them basic CSSCs (BCSSC) or fatigue related CSSCs as they can be obtained from the strain controlled fatigue tests by monitoring the material stress response. As this procedure is time consuming several shortcut approaches have been proposed using repeated block loading (the result is in the form of the macroblock cyclic stress-strain curve MCSSC), incremental step loading, etc. [2]. The results of these procedures may differ (especially for cyclically unstable materials) and may be influenced by mode control and mean values of loading [3].

Nevertheless, sinusoidal loading of mechanical systems is relatively rare in practice and the majority of operating loads possess a random character. The natural questions arising in this connection are therefore as follows:

– how to define the CSSC or its analogy for random load conditions (here designated as the random cyclic stress-strain curve RCSSC)?

– do the RCSSC parameters change with the number of load ordinates for cyclically unstable materials?

– is there a stabilized RCSSC to which the „evolutionary“ RCSSCs converge during loading of an unstable material?

– in case that such a stabilized RCSSC exists, what is its relation to the BCCSC and CSSC?

This kind of information is almost missing in the literature (some thoughts could be found in [4]) and the results described further try to clarify and answer these questions.

2. Experimental

Experiments were performed in the electrohydraulic computerized testing machine EDYZ 6 controlled by a single purpose software developed in our Institute.

The strain controlled tests were performed in the frequency range up to 10 cps. The load ordinates were provided at the repeated frequency of 600 cps; this was also the frequency at which deformation and stress histories were measured. The evaluation of energy dissipated in the unit material volume was continuously obtained by the on-line integration of (σ vs. ε) values and the results were stored on a disk together with the values corresponding to the local extremes σ_p and ε_p. The rate of loading could be varied during the test which appeared to be advantageous as in some cases the specimen surface temperature reached an unacceptable level. Thus the actual rate selected did not cause specimen heating above 70 °C which corresponded to the mean dissipated energy in the unit specimen volume up to 4 MPa/cycle.

The specimens were made of low carbon steel with ground surface and diameter of 10 mm.

As the system controlled and measured the total deformation ε, its plastic part ε_p was obtained by subtracting the elastic part, i.e. $\varepsilon_p = \varepsilon - \sigma/E$, where E is the modulus of elasticity.

The experimental aim was to obtain

- the BCSSC and its parameters k_B and n_B, and
- the MCSSC for various macroblock intensities and its parameters k_M and n_M.

Further, based on the experimental evidence of the cyclic stress-strain behaviour under random loading, we intended to formulate a definition of the random cyclic stress-strain curve (RCSSC), obtain its corresponding parameters k_R and n_R, and compare them with k_B, n_B, k_M, and n_M.

3. Results

3.1 Basic cyclic stress-strain curve (BCSSC) and macroblock cyclic stress-strain curve (MCSSC)

Fig. 1 shows the BCSSC as well as the MCSSCs for three macroblocks with different intensities (strain variances) S07, S08, and S09 (S07 was the lowest and S09 the highest intensity). The macroblock used contained 64 levels organized into a triangle with ascending-descending amplitudes and was repeatedly applied up to fracture. Most partial blocks contained up to 10^4 cycles which was sufficient to reach the saturation of mechanical properties. Even for the highest macroblock intensity (S09 specimen) the number of cycles to fracture was high ($2N_f = 366274$), which corresponds to the high cycle fatigue range.

It is obvious that the MCSSCs obtained are not influenced by the intensity of macroblock loading. Their course in the coordinates $\log \sigma_a - \log \varepsilon_{ap}$ can be split into three clearly different parts:

- for $\varepsilon_{ap} \leq 1 \times 10^{-5}$ the experimental points show a large scatter but these measurements (corresponding to very long lives) reach the limiting accuracy of the measuring system; for most practical applications this range does not represent an interest;
- in the range between $10^{-5} \leq \varepsilon_{ap} \leq 10^{-4}$ the experimental points form a straight line (power law function) but with a clearly different slope compared with the BCSSC;
- finally, in the range above $\varepsilon_{ap} \geq 10^{-4}$ both sets of points lie on one straight line but the BCSSC seems to show a larger scatter (Fig. 2); in this low-cycle fatigue range the coincidence of all curves is obviously very good.

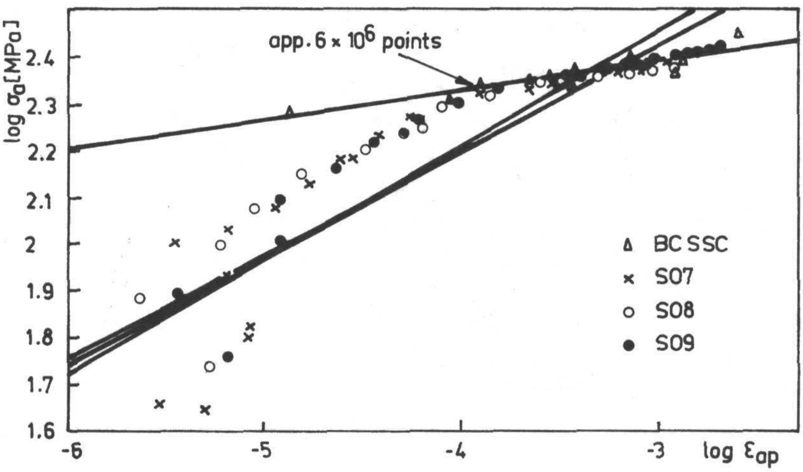

Fig. 1. Cyclic stress-strain curves for constant amplitude loading (BCSSC) and macroblock loading (MCSSC) with three different intensities S07, S08, and S09.

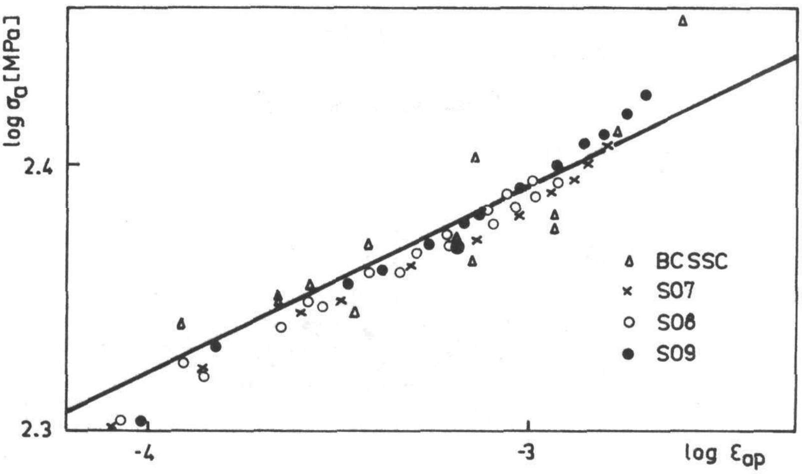

Fig. 2. Similar as in Fig. 1 but for points above $\varepsilon_{\mathrm{ap}} = 10^{-4}$.

3.2 Random cyclic stress-strain curve RCSSC

In the first step we have defined the RCSSC *as a set of centered closed hysteresis loop peaks* obtained by the Rain Flow Method. This was enabled by continuous

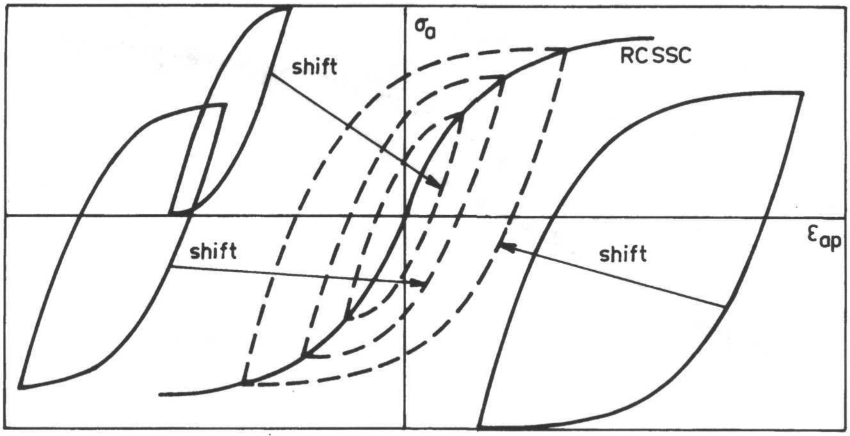

Fig. 3. Principle of determining random cyclic stress-strain curve (RCSSC) from centred closed loops.

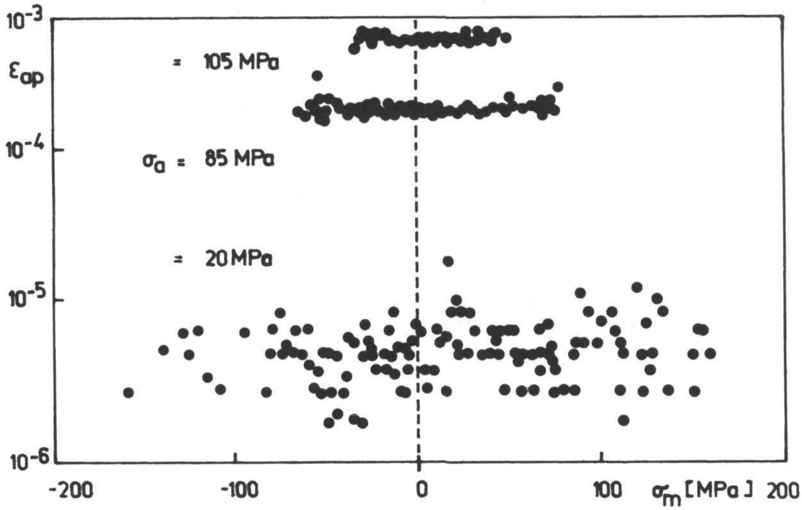

Fig. 4. Distributions of closed-loop mean stress levels σ_m for various closed-loop heights σ_a.

monitoring of the stress-strain history and detection of locations of the closed loops which were afterwards shifted to a common zero mean stress and zero mean strain

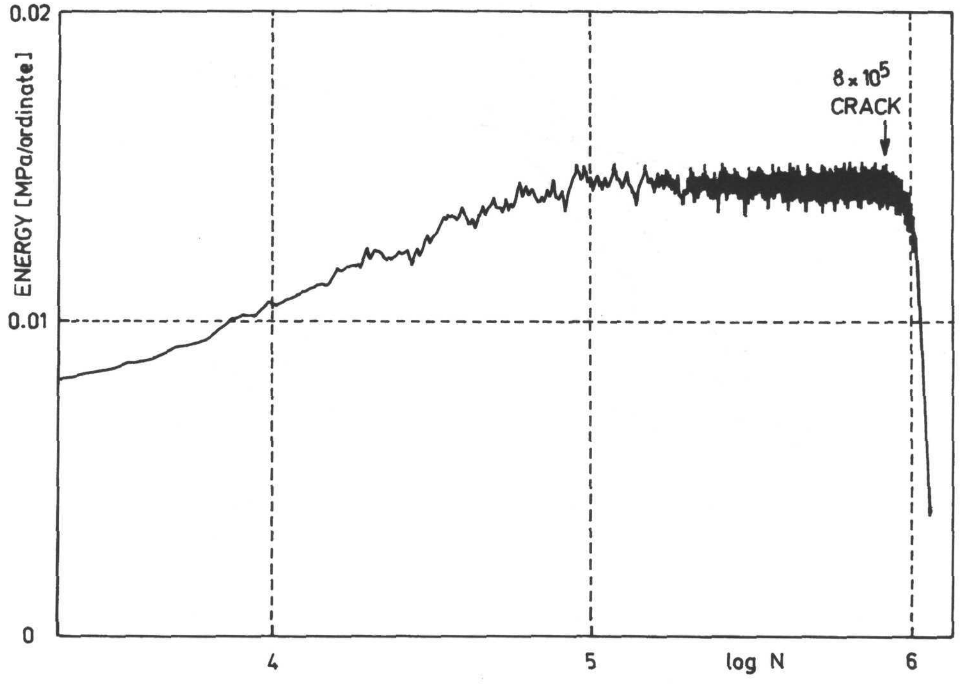

Fig. 5. Evolution of closed-loop areas (dissipated energy) under Gaussian white noise loading.

(Fig. 3). This idea of neglecting the corresponding loop mean levels obviously includes an implicit assumption that the closed-loop mean stress and mean strain are secondary factors only, which do not profoundly influence the fatigue damage accumulation process. Nevertheless, this point could be questionable as most Haigh's diagrams show negative effects of positive (tensile) mean stresses and positive effects of negative (compressive) mean stresses. Even if this is so, Fig. 4 brings an evidence that these two effects might be at least partially mutually balanced as the statistical distribution of the positive and negative closed-loop mean stresses σ_m for a given stress amplitude σ_a is practically uniform.

In order to determine the RCSSC parameters k_R and n_R, we have applied white noise strain random process (constant power spectral density) with the Gaussian distribution of ordinates and at the beginning monitored the closed-loop energy evolution (Fig. 5). From its dependence on the number of closed-loop peaks it has become obvious that

– the low carbon steel used is cyclic softening up to 10^5 peaks,

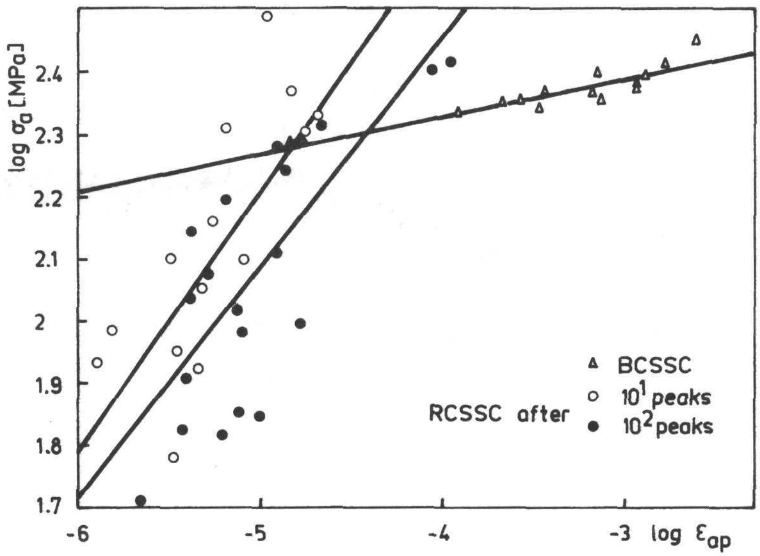

Fig. 6. Random cyclic stress-strain curves (RCSSCs) determined after 10^1 and 10^2 closed-loop peaks.

– around 10^5 peaks the cyclic mechanical properties are saturated and for rather a long time stay constant up to the crack initiation at about 8×10^5 peaks,

– when the crack initiates between extensometer edges the closed loops are distorted (bent) and the test looses its sense as the fatigued cross section is indeterminate.

Because our task was to follow the RCSSC evolution we have determined five curves corresponding to $10^1, 10^2, 10^3, 10^4$, and 10^5 closed-loop peaks (Figs. 6 and 7 in the coordinates $\log \sigma_a - \log \varepsilon_{ap}$). Fig. 6 illustrates that the RCSSCs corresponding to 10^1 and 10^2 peaks are relatively steep and with very large scatters due to the limiting accuracy of measuring. They characterize the beginning of material softening which is in accord with Polák's findings [2]. The correlation of these RCSSCs with the BCSSC is very poor, however.

The RCSSCs obtained after $10^3, 10^4$, and 10^5 peaks gradually change their position illustrating the material cyclic softening process. For smaller strain amplitudes (say, below $\varepsilon_{ap} < 10^{-4}$) their slope and position are far away from those of the BCSSC; the coincidence is slightly better for $\varepsilon_{ap} \geq 10^{-4}$ only (Fig. 8). Thus the results document that under random loading there is a strong correlation between the height (peak stress) and width (strain amplitude) of the closed loops, similarly as in the case of the BCSSC; nevertheless, these relations are different

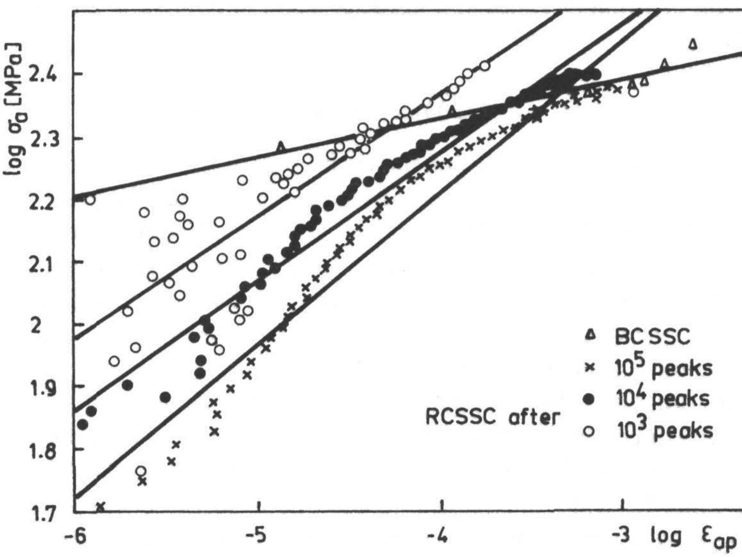

Fig. 7. The same as in Fig. 6 but for $10^3, 10^4$, and 10^5 peaks.

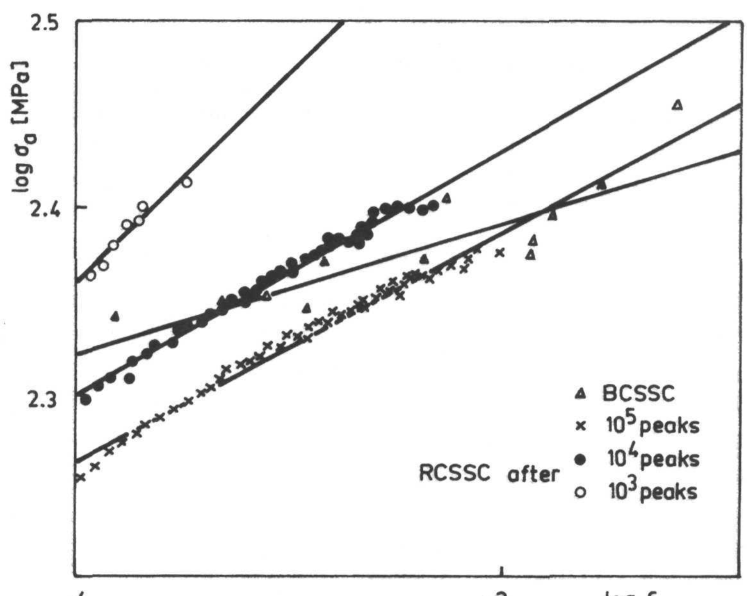

Fig. 8. The same as in Fig. 7 but for points above $\varepsilon_{ap} = 10^{-4}$.

from those obtained under sinusoidal loading. Their similarity is better for larger strain amplitudes.

4. Discussion

Table 1 summarizes the values of the cyclic stress-strain parameters obtained for the BCSSC, MCSSCs (three macroblock intensities) and RCSSCs (after five numbers of peak values $10^1, 10^2, 10^3, 10^4$, and 10^5) for all points obtained and for only $\varepsilon_{ap} \geq 1 \times 10^{-4}$. Fig. 9 illustrates these values graphically for the number of peaks above 10^3. One can see that both macroblock and basic methods of determination of the cyclic stress-strain parameters n and k yield practically identical results, on average $n = 0.081$ and $k = 431.2$ MPa.

Different values are obtained, however, for the RCSSCs. Even if the starting period of entirely unstable material behaviour up to 10^3 peaks is neglected, the n_R values are systematically higher ranging between 0.226 and 0.123. For the strain controlled loading it means that stresses computed from Eq. (1) are smaller using the RCSSC than using the BCSSC or MCSSC. Alternatively, under stress controlled conditions a higher n_R yields also a higher ε_{ap}.

Under strain control higher values of k obtained from the RCSSC generate higher stresses, whereas under stress control smaller deformations.

Table 1. Comparison of parameters of the cyclic stress-strain curves

Type of loading	Evaluated parameters from: all points		points corresponding to $\varepsilon_{ap} \geq 1 \times 10^{-4}$ only	
	k [MPa]	n	k [MPa]	n
Sinusoidal (strain control)	Basic cyclic stress-strain curve			
	377.1	0.0620	400.9	0.0704
Macroblock	Macroblock cyclic stress-strain curve			
S07	1564.2	0.2458	418.6	0.0776
S08	1237.9	0.2235	421.7	0.0795
S09	1268.1	0.2273	453.4	0.0871
Gaussian white noise	Random cyclic stress-strain curve after			
10^1 peaks	28597.3	0.4440	–	–
10^2 peaks	11558.7	0.4004	–	–
10^3 peaks	1407.2	0.1949	1826.6	0.2256
10^4 peaks	1230.0	0.2043	663.2	0.1304
10^5 peaks	1485.5	0.2410	571.7	0.1233

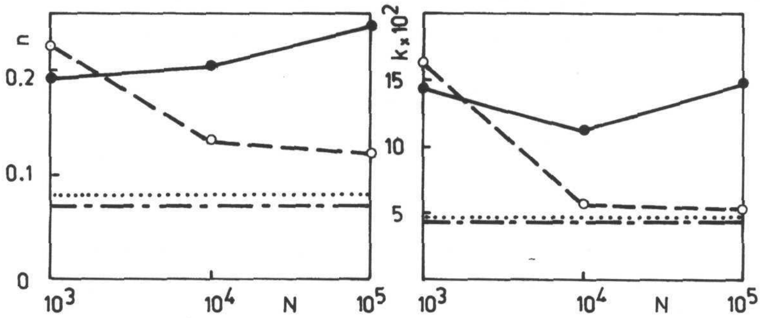

Fig. 9. Illustration of n and k parameters obtained under constant sinusoidal loading (BCSSC $- \cdot - \cdot -$), macroblock loading ($\cdots\cdots$), and Gaussian white noise loading (—— all points, $- - -$ points above $\varepsilon_{ap} = 10^{-4}$) for various numbers of peaks.

These facts evoke the natural question whether the use of the MCSSC or BCSSC is adequate because most materials are loaded randomly and so the use of the RCSSC should be more appropriate.

The answer is problematic, however. First of all, the presented results were obtained for one type of a random process only and so any generalization is, for the time being, impossible. Secondly, the higher values of k_R and n_R obtained for the RCSSCs may partially balance each other. This means that under strain control a higher n_R conditions smaller σ but a higher k_R leads to higher σ; under stress control a higher n_R brings about higher ε but a higher k_R gives smaller ε. Nevertheless, numerical verifications seem to yield comparable results for higher stress and strain amplitudes and also for higher numbers of process peaks. This can be deduced from Table 2.

Table 2. Stress and strain values obtained from cyclic stress-strain curve equation (1) for k and n parameters experimentally determined under sinusoidal loading (S), macroblock loading (M) and random loading (R), resp., after $10^3, 10^4$, and 10^5 process peaks

Strain		Stress σ [MPa] after		
		10^3 peaks	10^4 peaks	10^5 peaks
$\varepsilon_{ap} = 10^{-6}$	S	151.5	151.5	151.5
	M	140.0	140.0	140.0
	R	80.9	109.5	104.1
$\varepsilon_{ap} = 10^{-3}$	S	246.5	246.5	246.5
	M	245.6	245.6	245.6
	R	384.4	269.4	243.9

Further investigations for other materials and other random processes are required, however, before practical recommendations can be provided.

5. Conclusions

From the experimental results presented above the following conclusions can be drawn:

1. Under random loading it is possible to design a curve which resembles the material cyclic stress-strain curve. This curve represents a set of peaks of centered closed-loop hysteresis loops obtained by the Rain Flow Method and is called the random cyclic stress-strain curve (RCSSC).

2. During the process of saturation of the cyclic material mechanical properties the RCSSC also changes its slope and position. In our case of the cyclic softening low carbon steel both the RCSSC parameters k_R and n_R were higher compared with the parameters obtained under sinusoidal and macroblock loading. So the stable RCSSC (for a large number of peaks) seems to approach the basic cyclic stress-strain curve but for higher strain amplitudes only (above $\varepsilon_{\mathrm{ap}} = 10^{-3}$). For smaller strain amplitudes in the range $10^{-3} \leq \varepsilon_{\mathrm{ap}} \leq 10^{-6}$ the process of material cyclic softening causes that the RCSSC deviates more and more from the BCSSC.

3. The use of higher values of the parameters k_R and n_R corresponding to the RCSSC obtained under Gaussian white noise loading seems to be adequate for higher strain amplitudes (low cycle fatigue) but doubtful for smaller strain amplitudes. Further experimental evidence is required, however, to elucidate these relations for other materials with different cyclic properties.

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