EXPERIMENTAL VERIFICATION OF THERMODYNAMIC THEORY OF ELASTO-PLASTIC DEFORMATION

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The stress-strain curves were measured by the method of the uniaxial tension test. The theoretical stress-strain relation derived on the basis of generalized thermodynamics was tested by experiments. The material constants were determined from the best fitting of the experimental data with the stress-strain relation. It is shown that the material constants are temperature dependent.

Key words: elastic strain tensor, inelastic strain tensor, Cauchy stress tensor, Young's modulus, Poisson's ratio, yield stress

EXPERIMENTÁLNE OVERENIE TERMODYNAMICKEJ TEÓRIE ELASTICKO-PLASTICKEJ DEFORMÁCIE

Boli namerané napäťovo-deformačné krivky metódou jednoosového namáhania. Experimentálne bol otestovaný napäťovo-deformačný vzťah odvodený s použitím zovšeobecnenej termodynamiky. Modelové parametre boli určené optimálnym zosúladením teórie s experimentom. Bola zistená teplotná závislosť týchto parametrov.

1. Introduction

One of the distinct features of plastic deformation is its irreversibility. The plastic deformation represents an inherently irreversible process. In classical thermodynamics it is assumed that if the process is sufficiently slow it can be treated as a reversible one. This assumption is wrong in the case of the plastic deformation process, because no matter how slow the process is, the plastic deformation remains irreversible. From this it follows that the plastic deformation can be described only on the basis of non-equilibrium thermodynamics. The further distinct feature of plastic deformation is that the deformation process cannot be described by the

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inelastic strain tensor $\boldsymbol{\epsilon}_\mathrm{p}$ only. It is worth to mention that the total strain tensor is the sum of elastic and inelastic (plastic) part of strain tensor

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_\mathrm{e} + \boldsymbol{\epsilon}_\mathrm{p}, \tag{1}$$

where $\boldsymbol{\epsilon}_\mathrm{e}$ is the elastic part of strain tensor and $\boldsymbol{\epsilon}_\mathrm{p}$ is the plastic part of strain tensor.

In this paper, whenever the strain tensor becomes part of the development, we will be restricted to its first approximation, i.e. to the infinitesimal strain. Now, it is generally accepted that the internal state of a plastically deformed body is described not only by the standard state variables but also by the internal state variables. If extra state variables are introduced then we speak of extended or generalized thermodynamics. The internal state variables can be scalar (chemical reaction, relaxation phenomena and inelastic hardening) [1–4] and [5], vectors (dielectric and magnetic relaxation phenomena) [6–9] and tensors (plastic deformation) [10, 11] and [12].

In the case of plastic deformation the internal state variables describe, on the macroscopic level, the change of the dislocation arrangement. It is well known that the plastic deformation of metals and many other crystalline materials is accomplished through the motion of a line shaped crystal defects called the dislocations. The dislocation arrangement is determined by the distribution function. From the theory of probability it is known that the distribution function is equivalently defined by all its statistical moments. The statistical moments, on the macroscopic level, can play the role of internal state variables. According to this statement Drucker has surmised that the number of internal state variables should be infinite. But Kröner concludes in his paper [14]: "It is doubtless not possible to prepare two specimens of the same material which have exactly the same microscopic dislocation arrangement. Nevertheless, one observes a good reproductibility in many types of plastic experiments. From this it can be concluded almost with certainty that one does not need the high-order moments of the dislocation arrangement for a reasonable plasticity theory. How far one has to go in the order of the moments will depend on the experiment itself". In [4] it was assumed that the internal state of the solid is described by only one tensorial state variable. In the case of the simple uniaxial tension test, it is sufficient to consider a scalar internal state variable. Later on it will be shown that the experimental stress-strain curves can be interpreted by the theory, which uses only one scalar internal state variable.

2. Theory

In [5] on the basis of generalized thermodynamics, there were derived the following phenomenological equations:

$$\frac{\mathrm{d}\boldsymbol{\epsilon}_\mathrm{p}}{\mathrm{d}t} = \frac{K}{1+\eta_0}\boldsymbol{\sigma} + \frac{K}{1+\eta_0}\frac{\eta_0}{1-2\eta_0}\sigma\mathcal{I} - A\beta\mathcal{I} \tag{2}$$

and

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = A\sigma - B\beta, \tag{3}$$

where $\boldsymbol{\sigma}$ is the symmetric Cauchy stress tensor, α is the internal state variable, β is the affinity conjugate to the internal state variable α, K, η_0, A and B are the material constants, which will be determined from the experiment, and $\boldsymbol{\mathcal{I}}$ is the unit tensor.

Equations (2) and (3) describe the plastic deformation. The deformation process is composed of two regions. The first one is the elastic one in which the deformation process is described by the Hooke's law

$$\boldsymbol{\sigma} = \boldsymbol{C}^4 : \boldsymbol{\epsilon}_\mathrm{e}, \tag{4}$$

where $\boldsymbol{C}^4$ is a fourth order elastic compliance tensor. The second one is the elasto--plastic region. We will assume that in the second region the elastic and plastic deformation simultaneously take place. These two regions are separated by $\epsilon_{11\mathrm{c}}$ and $\sigma_{11\mathrm{c}}$ in the case of the uniaxial tension test. In the first region there may exist also a non-linear deformation process but we will not consider this type of deformation. The possibility of neglecting the non-linear deformation will be proved by comparison of theoretical and experimental results. In [5] from Eqs. (2) and (3) for the uniaxial tension test, where a circular cylinder was subjected to a uniform axial tension $\sigma_{11} \neq 0$ and all other $\sigma_{ij} = 0$, the following equation was derived:

$$\frac{\mathrm{d}^2\sigma_{11}}{\mathrm{d}t^2} + (B\nu_2 + EH)\frac{\mathrm{d}\sigma_{11}}{\mathrm{d}t} + E\nu_2(HB - A^2)\sigma_{11} = EB\gamma\nu_2, \tag{5}$$

where

$$\gamma = \frac{\mathrm{d}\epsilon_{11}}{\mathrm{d}t} = \text{const.}, \tag{6}$$

$$H = \frac{K}{1+\eta_0}\frac{1-\eta_0}{1-2\eta_0}, \tag{7}$$

ν_2 is the material constant and E is the Young's modulus. In this case the Hooke's law has the form

$$\sigma_{11} = E\epsilon_{11}. \tag{8}$$

The solution of Eq. (5) is as follows:

$$\sigma_{11}(t) = D + C_1\mathrm{e}^{\lambda_1(t-t_0)} + C_2\mathrm{e}^{\lambda_2(t-t_0)}, \tag{9}$$

where

$$\lambda_{1,2} = -\xi_1 \pm \xi_2, \tag{10}$$

$$D = \frac{B\gamma}{HB - A^2}, \tag{11}$$

$$\xi_1 = \frac{B\nu_2 + EH}{2}, \tag{12}$$

$$\xi_2 = \sqrt{\frac{1}{4}(B\nu_2 - EH)^2 + EA^2\nu_2}, \tag{13}$$

where t_0 is the time of beginning of plastic deformation. $\lambda_{1,2}$ are the roots of the characteristic equation

$$\lambda^2 + (B\nu_2 + EH)\lambda + E\nu_2(HB - A^2) = 0. \tag{14}$$

The constants C_1 and C_2 are determined from the initial conditions. For $t = t_0$, $\sigma_{11}(t_0) = \sigma_{11c}$ and from Hooke's law it follows

$$\left.\frac{\mathrm{d}\sigma_{11}}{\mathrm{d}t}\right|_{t=t_0} = E\gamma. \tag{15}$$

Applying these initial conditions we obtain

$$C_1 = \frac{1}{\lambda_1 - \lambda_2}E\gamma - \lambda_2(\sigma_{11c} - D), \tag{16}$$

$$C_2 = \frac{1}{\lambda_1 - \lambda_2}\lambda_1(\sigma_{11c} - D) - E\gamma. \tag{17}$$

Integrating $\frac{\mathrm{d}\epsilon_{11}}{\mathrm{d}t} = \gamma$ one obtains $\gamma(t - t_0) = \epsilon_{11} - \epsilon_{11c}$, where $\epsilon_{11c} = \epsilon_{11}(t_0)$. Introducing the last relation into (9) we obtain the stress-strain relation

$$\sigma_{11} = D + C_1 \mathrm{e}^{\lambda_1 \frac{\epsilon_{11} - \epsilon_{11c}}{\gamma}} + C_2 \mathrm{e}^{\lambda_2 \frac{\epsilon_{11} - \epsilon_{11c}}{\gamma}}. \tag{18}$$

Relation (18) will be tested by comparison with the experimental data.

3. Comparison of theoretical and experimental results

For the test of all assumptions, which have been made through the derivation of the stress-strain relation, we have measured the stress-strain curves of some chosen alloyed steels. The measurements were done on the tensile testing machine Schenk PLX using the method of the uniaxial tension test. The permanent elongation was chosen $\gamma = 10^{-5}\text{mm}\cdot\text{s}^{-1}$. The initial length of all samples was $l_0 = 40$ mm. The samples had the circular form and were subjected to a uniform axial tension, $\sigma_{11} \neq 0$ and other $\sigma_{ij} = 0$. All experiments were done at the condition that $\frac{\text{d}\epsilon_{11}}{\text{d}t} = \gamma$, where $\gamma = \frac{1}{l_0}\frac{\text{d}l}{\text{d}t} = 10^{-6}\ \text{s}^{-1}$.

The composition of the individual steels is presented in Table 1. The samples 1 and 3 were heat-treated but sample 2 was not. The relevant values of the material constants are presented in Table 2.

Table 1

Sample Nr.	1, 2	3
Type of steel	15CH2NMFA	STN15121
C [wt.%]	0.13÷0.18	0.1÷0.18
Mn [wt.%]		0.4÷0.7
Mo [wt.%]	0.5÷0.7	0.4÷0.6
Ni [wt.%]	1.0÷1.5	
Cr [wt.%]	1.8÷2.30	0.7÷1.3
Si [wt.%]	0.17÷0.37	0.15÷0.35
P, S [wt.%]	max. 0.02	max. 0.04
Sn [wt.%]	max. 0.3	
V [wt.%]	0.1÷0.12	
As [wt.%]	max 0.04	

From the best fitting of the experimental data with relation (18) we have obtained the values of the parameters $C_1, C_2, \lambda_1, \lambda_2, \sigma_{11c}$, and E, which are presented in Table 2. With the help of these parameters we have calculated the material constants $D, H, B\nu_2$, and $A^2\nu_2$ according to relations

$$D = \sigma_{11c} - C_1 - C_2, \tag{19}$$

$$H = -\frac{D}{\gamma E^2}\lambda_1\lambda_2 - \frac{\lambda_1 + \lambda_2}{E}, \tag{20}$$

Table 2

Sample Nr.	1		2		3	
T [°C]	20	500	20	500	20	500
$\lambda_1 \times 10^3$ [s^{-1}]	-0.340	−0.237	−1.462	−0.936	−0.296	−0.787
$\lambda_2 \times 10^5$ [s^{-1}]	−16.14	−21.29	−5.389	−18.34	−0.6273	−14.62
C_1 [MPa]	−181.0	−143.5	−133.8	−173.6	−648.6	−161.4
C_2 [MPa]	−799.3	−551.5	−33.82	−41.92	−296.0	−298.7
σ_{11c} [MPa]	280.9	321.1	470.4	299.4	277.7	385.4
$E \times 10^{-5}$ [MPa]	1.906	1.515	1.974	1.702	1.911	1.706
$H \times 10^9$ [MPa$^{-1}\cdot$s^{-1}]	0.725	0.135	6.390	3.526	1.542	2.129
$B\nu_2 \times 10^5$ [s^{-1}]	36.34	33.87	25.46	51.95	0.7634	56.98
$A^2\nu_2 \times 10^{13}$ [MPa$^{-1}\cdot$s^{-2}]	0.2458	0.8437	12.28	8.231	0.02054	5.388

$$B\nu_2 = -(\lambda_1 + \lambda_2) - EH, \tag{21}$$

and

$$A^2\nu_2 = B\nu_2(H - \frac{\gamma}{D}). \tag{22}$$

These relations were derived with the help of relations (7), (8), (9), (10), (11), (12), (16) and (17).

The values of the parameters $D, H, B\nu_2$, and $A^2\nu_2$ are presented in Table 2. The stress-strain curves of 15CH2NMFA (heat-treated), 15CH2NMFA (not heat-treated) and STN15121 (heat-treated) are depicted in Figs. 1, 2 and 3, respectively. The measurements were done at temperatures 20 °C and 500 °C. The temperature 500 °C was reached after 45 min. The sample was kept at 500 °C 15 min before the experiment. From the Figs. 1, 2 and 3 it is seen that the theoretical stress-strain relation describes quite well the experimental stress-strain curves. The material constants are dependent on the temperature. The Young's moduli decrease with increasing temperature. In our model the threshold stress σ_{11c}, at which the plastic deformation begins, has an interest-

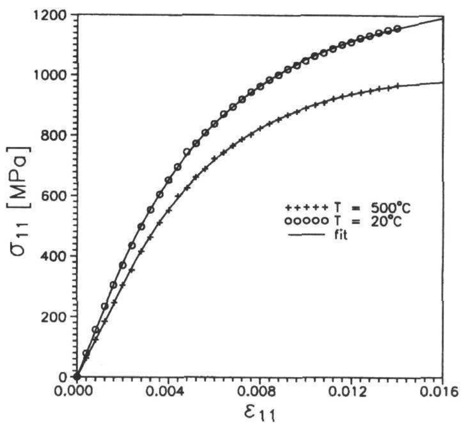

Fig. 1. Stress-strain curves of sample 1 (heat-treated steel).

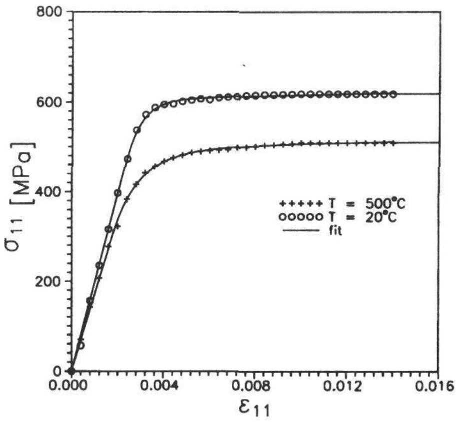

Fig. 2. Stress-strain curves of sample 2 (heat-untreated steel).

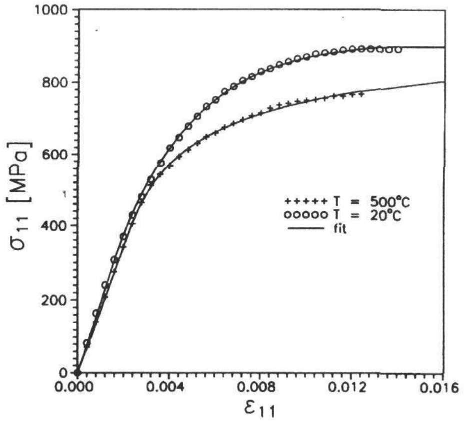

Fig. 3. Stress-strain curves of sample 3 (heat-treated steel).

ing temperature behaviour. Plastic deformation observable at the macroscopic level results from the motion of dislocations through the material. The motion of dislocations takes place in the slip-planes. Different slip-planes have a different orientation and, therefore, in the case of the uniaxial tension test the stress in the slip-planes is different. There is a certain slip-plane, the orientation of which is optimal according to the external tensile load. In this slip-plane there is the largest stress compared to the other slip-planes. When an external force is gradually applied, the stress in the optimal slip-plane will be the first one which gets the threshold value. So from this moment the plastic deformation begins. If the heights of barriers occur in a large range, stress-strain curves are steep, so this is the case of heat-treated material as it is seen from Figs. 1 and 3. But when the range is narrow the stress-strain curves are flat as it is seen in Fig. 2. From the Table 2 it is seen that $\sigma_{11c} = 470$ MPa in the case of a not heat-treated steel and $\sigma_{11c} = 280$ MPa in the case of the heat-treated steel. From this fact we can conclude that the minimal height of the barrier in the optimal slip-plane in the case of the not heat-treated steel is 1.7 times higher than in the case of the heat-treated steel. With the increasing temperature the heights of barriers decrease. Due to this fact, the stress-strain curves are situated lower at 500 °C than at 20 °C. But the situation with the minimal value of the height of the barrier in the cases of not heat-treated steel differs from the heat-treated ones. As mentioned above, the minimal value of the height of barrier is 1.7 times higher in the case of not heat-treated steel than in the case of heat-treated one. Therefore in the case of not heat-treated steel all heights of barriers decrease with increasing temperature and, therefore, σ_{11c} decreases. But in the case of heat-treated steel the barriers of minimal values of the heights gradually disappear with increasing temperature and, therefore, σ_{11c} increases with increasing temperature. The similar behaviour of the threshold stress was observed also in [5]. Generally, it is accepted that the yield stress decreases with the increasing temperature. The yield stress is defined at the 0.2% of the permanent ϵ_{11}. From the Table 3 it is seen that the above statement is fulfilled.

Table 3

Type of material	Temperature [°C]	Yield stress [MPa]
15121	20	734
	500	656
15CH2NMFA heat treatment	20	872
	500	784
15SH2NMFA without heat treatment	20	603
	500	484

4. Conclusions

– The theoretical stress-strain relation was tested by experiments.

– The stress-strain curves were measured on CrMo steels.

– From the best fitting of the stress-strain curves the values of the material constants were determined.

– It was found that the material constants depend on the temperature.

– The threshold stress at the material without heat treatment decreases with increasing temperature. On the contrary, the threshold stress at the material with heat treatment increases with increasing temperature.

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