A SIMPLE PHENOMENOLOGICAL HYPOTHESIS OF CUMULATIVE FATIGUE DAMAGE PART II. GENERAL FORMULATION AND DISCUSSION

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The notion of the history damage factor introduced in the first part of this paper poses problems when loading half-cycles interrupted by smaller cycles are considered. Consequent formulation of the hypothesis solving these problems for general complex loading is presented for the integrity of the theory. General formulation of the proposed hypothesis preserves its simplicity. Physical aspects of the hypothesis are briefly discussed and some ideas for further research presented.

Key words: cumulative fatigue damage, hypothesis, loading sequence, complex loading

JEDNODUCHÁ FENOMENOLOGICKÁ HYPOTÉZA KUMULÁCIE ÚNAVOVÉHO POŠKODENIA ČASŤ II. VŠEOBECNÁ FORMULÁCIA A DISKUSIA

Pojem faktor poškodenia zohľadňujúci históriu zaťažovania, zavedený v prvej časti tohto článku, spôsobuje problémy, keď treba uvážiť zaťažovacie polcykly prerušené inými menšími cyklami. Pre ucelenosť teórie sa uvádza dôsledná formulácia hypotézy, ktorá rieši tieto problémy pre prípad všeobecného zložitého zaťažovania. Všeobecná formulácia navrhovanej hypotézy zachováva jej jednoduchosť. Stručne sa diskutuje o fyzikálnych aspektoch hypotézy a predkladajú sa niektoré návrhy na ďalší výskum.

1. Introduction

In the first part of our paper [1], we have considered only whole loading half-cycles in the hypothesis formulation. In the complex loading case, however, half-cycles defined by the rainflow method (e.g. half-cycle $M = 1$, in Fig. 1) are interrupted by smaller ones ($M = 2$, $M = 3$). In this case two basic questions arise. First, does the part of the interrupted half-cycle before interruption (e.g. $M = 1$,

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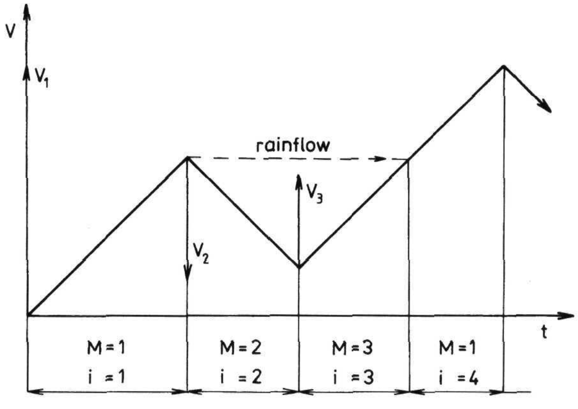

Fig. 1. Example of loading and rainflow definition of half-cycles (numbered by M), definition of loading sections (numbered by i), and definition of relative damage controlling quantities (V_M).

in loading section $i = 1$ in Fig. 1) create corresponding part of the history damage factor when interrupting half-cycles ($M = 2$, $M = 3$) are running? Second, do the interrupting half-cycles create corresponding part of the history damage factor when the rest of the interrupted half-cycle ($M = 1$, $i = 4$) is running? It is obvious that answers to both these questions are affirmative and our hypothesis should solve them in some way.

The aim of this paper is to provide consequent general formulation of our hypothesis, briefly discuss its physical aspects and present some ideas for the further research.

2. General formulation of the hypothesis

Consequent application of the notion of the history damage factor in complex loading case forces us to abandon the "whole half-cycle" approach. It seems that this is suitable always when the loading sequence is to be considered properly [2]. General formulation of our hypothesis consists in extrapolation of the whole half--cycle approach to the continuous form. In the whole half-cycle approach, only those points of loading are considered in which the half-cycles end. Values of damage controlling quantity and damage are determined by using these points. In general formulation of our hypothesis, all the points of loading are considered, i.e. loading is used like a continuous function of time, and damage is evaluated continuously. From the physical point of view, the damage mechanism of loading half-cycle is not known sufficiently to enable such continuous damage evaluation.

Therefore, our approach is an extrapolation of damage behaviour obtained by the whole half-cycle, or better the whole cycle, approach to the continuous form. This extrapolation is, of course, only an engineering approximation.

Hereafter, half-cycles defined by the rainflow method are considered. Continuous damage controlling quantity V (e.g. plastic strain) is supposed to be composed of damage controlling quantities, separately defined in each half-cycle, termed relative damage controlling quantities. Relative damage controlling quantity is a continuous non-negative quantity with zero value in the beginning of the half-cycle, in which it is defined, and with increment equal to the absolute value of increment of the damage controlling quantity V. Cyclic deformation behaviour of materials supports this assumption. Namely, it is well known that ascending and descending hysteresis loop branches are very similar, if not identical, in relative co-ordinate systems of relative stresses and relative strains [3, 4] defined for each branch of hysteresis loop separately. Relative damage controlling quantity can be identified with some relative quantity (e.g. relative plastic strain) defined for corresponding branch of hysteresis loop. Let us emphasise that the damage controlling quantity is not the quantity of the "range" type as it was in the half-cycle formulation. Relative damage controlling quantity in the M-th half-cycle is denoted by V_M.

Let us consider the time-behaviour of the loading divided into time intervals or loading sections. Loading section is the largest possible part of loading, in which one half-cycle or its part is performed continuously (Fig. 1). Obviously, if a half-cycle is interrupted, it is performed in several loading sections. Let i be the number of a loading section. Then M, the number of a half-cycle, may be formally considered to be a discrete function of the discrete argument i, $M = M(i)$. This will be emphasized by writing $V_{M(i)}$ for the relative damage controlling quantity. Half-cycles are numbered according to their beginnings, i.e. the half-cycle which begins before another one has lower number M than the one beginning later. This is important when a half-cycle is interrupted by smaller ones.

Increment of damage in an arbitrary point of an arbitrary half-cycle is defined by relation

$$\mathrm{d}D = H\mathrm{d}I. \tag{1}$$

Immediate damage factor I is defined as a continuous increasing function with zero value in the beginning of loading and with the increment given by

$$\mathrm{d}I(t) = \mathrm{d}\left[V^{\alpha}_{M(i)}(t)\right], \tag{2}$$

where t is an arbitrary time moment, i is the number of loading section, in which the considered time moment t is situated, and α is constant [1]. History damage factor H is defined as a continuous increasing function with zero value in the beginning of loading and with the increment

$$\mathrm{d}H(t) = \mathrm{d}\left[V_{M(i)}^{\beta}(t)\right], \tag{3}$$

where β is constant [1]. Similarly, like in the half-cycle formulation, at fracture it holds

$$D_\mathrm{f} = \int_0^{I_\mathrm{f}} H\mathrm{d}I = K, \tag{4}$$

where I_f is the immediate damage factor at fracture and K is constant [1]. Relation (4) consequently follows the time point of view, i.e. in any instant, damage can be determined regardless whether the current half-cycle or any of the previous ones are complete or not. This relation is suitable for general analyses, e.g. in the case of repeated block loading.

Integrating (3) from the beginning to the given instant, the history damage factor is obtained

$$H(t) = \sum_{j=1}^{i-1}\left(V_{M(j),\mathrm{end},j}^{\beta} - V_{M(j),\mathrm{beg},j}^{\beta}\right) + V_{M(i)}^{\beta}(t) - V_{M(i),\mathrm{beg},i}^{\beta}, \tag{5}$$

where $V_{M(x),\mathrm{beg},x}$ and $V_{M(x),\mathrm{end},x}$ are values of V_M in the beginning and in the end of x-th loading section, respectively, and j is the number of loading section in the previous loading history. Substituting (5) into (4) and integrating, it is obtained

$$\sum_{i=1}^{s_\mathrm{f}}\left\{\left[\sum_{j=1}^{i-1}\left(V_{M(j),\mathrm{end},j}^{\beta} - V_{M(j),\mathrm{beg},j}^{\beta}\right) - V_{M(i),\mathrm{beg},i}^{\beta}\right]\left(V_{M(i),\mathrm{end},i}^{\alpha} - V_{M(i),\mathrm{beg},i}^{\alpha}\right) + \right.$$
$$\left. + \frac{\alpha}{\beta+\alpha}\left(V_{M(i),\mathrm{end},i}^{\beta+\alpha} - V_{M(i),\mathrm{beg},i}^{\beta+\alpha}\right)\right\} = K, \tag{6}$$

where s_f is the number of loading section in the fracture instant. This relation enables counting the damage (left side) loading section by loading section.

In most cases, in the history damage factor, it is possible with sufficient accuracy to replace the contribution of just running loading section, i.e. the last two terms in relation (5), by the constant $\left(V_{M(i),\mathrm{end},i}^{\beta} - V_{M(i),\mathrm{beg},i}^{\beta}\right)$, i.e. by the effect of the whole just running loading section. Then, (6) is simplified to

$$\sum_{i=1}^{s_\mathrm{f}}\left[\sum_{j=1}^{i}\left(V_{M(j),\mathrm{end},j}^{\beta} - V_{M(j),\mathrm{beg},j}^{\beta}\right)\right]\left(V_{M(i),\mathrm{end},i}^{\alpha} - V_{M(i),\mathrm{beg},i}^{\alpha}\right) = K. \tag{7}$$

It is obvious that relation (7) is more conservative than relation (6) in any case. Let us note that the discussed simplification is only a mathematical one without any physical meaning. Similarly, when the last two terms in (5) are replaced by the one half of the constant used in the preceding case, i.e. by $\left(V^{\beta}_{M(i),\mathrm{end},i} - V^{\beta}_{M(i),\mathrm{beg},i}\right)/2$, then it is obtained

$$\sum_{i=1}^{s_\mathrm{f}} \left[\sum_{j=1}^{i-1} \left(V^{\beta}_{M(j),\mathrm{end},j} - V^{\beta}_{M(j),\mathrm{beg},j}\right) + \frac{1}{2} \left(V^{\beta}_{M(i),\mathrm{end},i} - V^{\beta}_{M(i),\mathrm{beg},i}\right) \right] \times \\ \times \left(V^{\alpha}_{M(i),\mathrm{end},i} - V^{\alpha}_{M(i),\mathrm{beg},i}\right) = K, \tag{8}$$

from which relation (I-23) follows. (We denote relations from the first part of our paper [1] by "I" preceding their number.) Namely, neglecting transient parts of loading half-cycles between loading levels in the l-level loading case, all beginning values of the relative damage controlling quantities in the loading sections are zero, all the end values of the relative damage controlling quantities on the given loading level are the same and equal to their range ΔV_i, and the number of loading sections is equal to the number of half-cycles. Using equation (I-9) and expressing the outcome in the matrix form, relation (I-23) is obtained.

Relation (8) corresponds with sufficient accuracy to (6) in the case of one-level and two-step loadings, excluding extremely short fatigue lives. Then, constant K in relation (6) follows from equation (I–9) and constants α and β follow from η and ϑ (I-18), (I-19).

Now, let us present another kind of simplification. Consider the history damage factor (5) at the end of M-th half-cycle H_M. Into H_M, let us include all whole half-cycles begun before the end of the M-th half-cycle regardless whether they are completed before the end of the M-th half-cycle or not. Then it holds

$$H_M = \sum_{M_\mathrm{h}=1}^{M} \Delta V^{\beta}_{M_\mathrm{h}}, \tag{9}$$

where M_h is the number of a half-cycle in the loading history. Remember that half-cycles are numbered according to their beginnings. It is obvious that the history damage factor (9) is greater or equal to the history damage factor (5) at the end of the M-th half-cycle. Further, let us suppose that the history damage factor (9) is applied to all loading sections of M-th half-cycle. Then, summing damage of all half-cycles, we have

$$\sum_{M=1}^{M_\mathrm{f}} \left[\sum_{M_\mathrm{h}=1}^{M} \Delta V^{\beta}_{M_\mathrm{h}} \right] \Delta V^{\alpha}_{M} = K, \tag{10}$$

where M_f is the number of the half-cycle at fracture. From our consideration it is clear that relation (10) is always more conservative than relation (6). It appears that quantitative difference between relations (10) and (6) is negligible in most cases. Relation (10) is simpler than relation (6). However, (10) does not consequently follow the time point of view, and thus, for theoretical analyses relations (6) or (4) are more convenient.

Finally, let us note that from relations (6)–(8), which are essentially composed of sums and products, the possibility of their good analysis can be assumed. Special, simple relation can be obtained for the number of block to fracture in the case of repeated block loading, however, such detailed analyses are beyond the scope of this work.

3. Discussion on physical aspects of the hypothesis

Our hypothesis introduces two new phenomenological notions, the immediate damage factor and the history damage factor, in addition to phenomenological notion of fatigue damage. Essentially, the notion of immediate damage factor is not completely new, because immediate effect of loading half-cycle (cycle) is considered in all fatigue damage hypothesis. The notion of "immediate damage factor" is introduced in correlation to the notion of "history damage factor". From the physical point of view, introduction of additional phenomenological notions, mainly of the history damage factor, complicates physical interpretation of our hypothesis. Despite this, separation of the immediate and history damage factors in our hypothesis appears to have physical reason. Indeed, it can be considered that cyclic loading causes not only fatigue damage, but the material reaction to the cyclic loading is more complex. Consequently, the fatigue damage can be influenced by another quantity which can represent the material reaction to preceding loading. Definition (I-1) is constructed as simple as possible and it supposes that the immediate effect of half-cycle is multiplied by the history effects. More detailed physical interpretation of our hypothesis is conditioned by physical interpretation of the history damage factor.

First, let us consider interpretation of the history damage factor (I-4), (5) as an effect of state of the whole material volume (represented by corresponding mechanical properties of material) on damaging in the critical point of material. The history damage factor is a linear function of the number of loading half-cycles $2N$ at one-level loading, $H = (2K)^{(1+\vartheta)/2}(2N_f)^{-\vartheta}(2N/2N_f)$. It is well known that the time-behaviour of mechanical properties of materials is strongly non-linear at cyclic loading [5], and so the history damage factor can be hardly derived from them. Of course, the state of the whole material volume can contribute to the history damage factor, however, also other material factors are required for its full physical interpretation. The state of surface layer differs from the state of the bulk of material and has substantial importance in the fatigue crack initiation and early crack growth stages [6]. In these stages, it would be possible to consider that development of persistent slip bands and short fatigue cracks and related phenomena are not fatigue damage exclusively, but also the physical basis of the history damage factor. Namely, fatigue damage can be identified with initiation and propagation of main fatigue crack. The main crack initiation and early growth are influenced by system of persistent slip bands and other cracks rising at the surface [7–9]. Synthesis of these influences could be identified with the history damage factor. However, the main crack also influences the other ones, and so it makes one system with them. Because of this, already in the early crack growth stage, exclusive position of the main crack in the system of all cracks is required for identification and separation of the history damage factor and damage in such a way. The same holds in the persistent slip band development and crack initiation stage. Of course, such interpretation of the history damage factor is complicated and further research is needed for its clarification. Interesting measurements are presented in [6]. Surface layer stress during cyclic loading with constant stress amplitude was determined for three materials. The surface layer stress σ_S was found to be linear function of the number of applied cycles, $\sigma_S = SN$, where the slope S depends on the stress amplitude. Expressing the stress amplitude by means of corresponding number of cycles to fatigue fracture, it can be written as $S = kN_f^p$, where k is a constant and p is an exponent. The surface layer stress σ_S is a characteristic of the surface layer state and could be identified with the history damage factor, which has the "slope" $\partial H/\partial N = (2K)^{(1+\vartheta)/2}2^{-\vartheta}N_f^{-(1+\vartheta)}$ at constant amplitude loading. It follows from data in [6] that exponent p has the value very close to minus one for all examined materials. Both slopes must be equal, and consequently ϑ must be zero. For $\vartheta = 0$, our hypothesis reduces to the linear one, and so the history damage factor can not be interpreted in such a way, in general. Nevertheless, quantities like surface layer stress σ_S are interesting for clarification of physical background of the history damage factor. Let us note that σ_S is correlated with damage in [6]. Now, let us take into account only one crack without interactions with the other ones in the fatigue short crack and fatigue crack propagation stages. It is well known that crack closure effect, due to permanent residual plastic deformations in the wake of advancing crack, influences the crack propagation in these stages [10]. Effective stress-intensity factor range proportional to $S_{max} - S_o$, where S_{max} is the maximum stress and S_o is the crack-opening stress, is used in the crack propagation law instead of stress-intensity factor [10]. Crack-opening stress S_o depends on loading history [11]. The maximum stress S_{max} represents the immediate effect of a half-cycle and the crack-opening stress S_o represents the history effect in our terminology. In the damage model based on effective stress-intensity factor range, however, S_{max} and S_o are not factors, i.e. they are not in a multiplicative relation defining the damage (crack length) increment, as it is postulated in our hypothesis (I-1). Thus, our hypothesis can be only in a very complicated relation to the history effects due to the crack closure. It appears that crack closure cannot explain crack growth behaviour for extremely small cracks and a crack tip zone must be taken into account [12, 13]. We could discuss the correspondence of model (I-1)–(I-6) with the relation between the state of crack tip zone and damaging. However, because the fatigue crack propagates, the state of crack tip zone can hardly represent the effect of the whole preceding loading, required by the history damage factor (I-4). Nevertheless, the state of crack tip zone can be one of several factors contributing to the history damage factor. As for the relation between the history damage factor and damage, it is to be noted that modification of history damage factor (I-4), (5) by means of multiplication by an arbitrary function of damage has no effect on evaluated fatigue lives. This brings to our hypothesis a more general meaning. However, if the supposed effect of the history damage factor is the solely manifestation of the preceding damage itself and magnitude of loading [14], physical adequacy of our hypothesis is dubious.

Physical interpretation of the history damage factor on the basis of contemporary knowledge about fatigue damage appears to be difficult. It is to be searched in the synthesis of effects of several material factors. Obviously, their final effect will be complicated. However, if this effect is linearizable with sufficient accuracy and depends on the whole preceding history, then the history damage factor has physical substantiation. The relation between history effects, immediate effect of half-cycle and damage increment is to be expected complicated, as well. Nevertheless, if damage increment equals with sufficient accuracy to the product of the history damage factor and immediate damage factor, then our hypothesis has a physical meaning.

4. Suggestions for further research

In the formulation of our hypothesis we have not explicitly dealt with the influence of mean stress value. One possibility of solving this problem is to take into account the effect of mean stress value in the definition of the damage controlling quantity. For instance, the damage controlling quantity ΔV can be identified with some of damage parameters from the work [15], which take into account the effect of mean stress value. Further derivation of relation (I-23) does not differ from the case of symmetrical loading, and so (I-23) holds also for the loading with asymmetrical loading levels in this case. Of course, the fatigue lives corresponding to given mean stress value are to be considered in relations (I-14) and (I-15). There is a slightly more complicated situation in the case of complex loading. In this case, the relative damage controlling quantities are supposed to be continuous, while known damage parameters are defined using discrete characteristics of the cycle [15]. Surely, these damage parameters can be extended to the compatible meaningful continuous form, too.

However, there is one principle question about the character of mean value effect, which is much more complicated. Has this effect the character of immediate damage factor only, or of history damage factor only, and particularly of history damage factor of cumulative type (I-4) and (5), or of both, as it was implicitly supposed in preceding discussion? The question is especially complicated in the fatigue crack initiation stage. Experiments with varying mean stress value could bring necessary knowledge in this field. However, such experiments are rare in literature [16, 17] and so, more generally valid conclusions are premature.

Another open question is the one of taking into account specificities of damaging near the fatigue limit. It can be simply solved by considering $\Delta V - \Delta V_C$, where ΔV_C is the value of damage controlling quantity at the fatigue limit, instead of the damage controlling quantity ΔV. A question arises, if the threshold value ΔV_C is to be considered only in the immediate damage factor, or in the history damage factor, too. If the threshold value is considered in both factors, then a question arises, whether it is possible to use in both of them the same value ΔV_C. The idea of two concurring processes working in the material better corresponds to fatigue limit understood as the phenomenon, at which it comes to non-propagation of a short crack. This idea opens some possibility to take into account the damage below the fatigue limit, too. The basic idea of our hypothesis enables its extension in this direction. However, such extensions lead to relatively more complicated models of damaging.

5. Conclusions

1. General formulation of cumulative fatigue damage hypothesis based on immediate and history damage factors is proposed for a general complex loading case. General formulation consequently follows the time point of view, i.e. in any time instant the history damage factor and damage can be determined regardless whether half-cycles are interrupted by smaller ones or not.

2. General formulation of proposed hypothesis keeps its simplicity and possibility of good analysis.

3. The half-cycle formulation of our hypothesis presented in the first part of our paper is a mathematical simplification of the general formulation of hypothesis.

4. Physical interpretation of the history damage factor and, as a result, the physical interpretation of our hypothesis is complicated. The physical interpretation of the history damage factor is to be searched in the synthesis of the effects of all relevant material factors.

Acknowledgements

This work was financed by means of the project VEGA 95/5305/634.

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Received: 8.1.1999
Revised: 2.6.1999