PROPAGATION OF PLASTIC INSTABILITIES

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The microscopic cause of the unstable deformation regime known as Portevin-LeChâtelier effect is the pronounced dynamic strain ageing when diffusing solute atoms modify the glide resistance in a specific manner. We attempt to describe the propagation of corresponding macroscopic deformation bands on the basis of a relaxation model that distinguishes between quasistationary and immediate flow resistances and incorporates non-local contributions to the flow stress.

1. Introduction

Plastic instabilities may occur for various reasons. We will focus on strain-rate softening instabilities that result from interaction of glide dislocations with diffusing solute atoms. In such regime of dynamic strain ageing (DSA) the locally variant solute distribution is formed, most probably by pipe diffusion, in the internal stress field or as a result of solute drag by moving dislocations [1]. The DSA-effects are observed for a lot of alloys deformed at suitable (medium) temperatures and strain rates.

If, as a consequence of DSA, the strain rate sensitivity of the flow stress becomes negative, significant spatial and temporal plastic instabilities and load serrations known as Portevin-LeChâtelier effect (PLC) may arise. Their main characteristic is propagation of continuous and hopping plastic waves or bursts. The aim of this work is to discuss the conditions of propagation of PLC-instabilities.

2. Quasistationary description of DSA

The glide resistance to movement of dislocations is produced by two sets of obstacles: strong forest dislocations and weak solute atoms. The dislocation component σ_d is given by

$$\sigma_d = \frac{F}{bL(\rho)} \approx \frac{F\sqrt{\rho}}{b}, \tag{1}$$

where F is the mean pinning force between the forest and waiting dislocations, L is the mean distance between obstacles along the waiting dislocation, ρ is the density

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of forest dislocations, b is the Burgers vector. The friction component σ_f, due to the mobile-dislocation-solute interaction, is expressed in single obstacle description by

$$\sigma_f = \frac{f}{bl(c)}, \tag{2}$$

where f and l have an analogical meaning as above but for solutes in the role of obstacles. c is the solute concentration.

The strain rate over only forest dislocations would be

$$\dot{\varepsilon} = \frac{b\rho_m \Lambda(\rho)}{t_w} = \Omega(\rho_m, \rho)\nu_0 e^{-\frac{\Delta G(F,c)}{kT}}, \tag{3}$$

where Λ is the mean free path of dislocations, t_w is the mean waiting time for a successful activation, ρ_m is the density of mobile dislocations, $\Omega \equiv b\rho_m\Lambda$ is the elementary strain, ν_0 is the attack frequency, ΔG is the activation free enthalpy, k is the Boltzmann constant, T is absolute temperature. A relation analogous to (3) should be valid for plastic flow in the field of solutes

$$\dot{\varepsilon} = \omega(\rho_m, c)\nu_0 e^{-\frac{\Delta g(f)}{kT}}. \tag{4}$$

Two qualitative differences between dislocation and friction kinetic equations (3) and (4) should be noted: 1. while the pre-exponential factor (elementary strain) for the flow rate across the dislocation forest depends only on the complete dislocation structure, that for the friction controlled rate varies with the (local line) concentration near the moving dislocation; 2. whereas the activation free enthalpy for the solute-dislocation interaction is a single obstacle characteristic, that for dislocation-dislocation interaction may be influenced by the presence of solutes (near the junction point).

The pairs of equations (1), (3) and (2), (4) yield the constitutive expressions $\sigma_d(\dot{\varepsilon}, T, \rho, \rho_m, c)$ and $\sigma_f(\dot{\varepsilon}, T, \rho_m, c)$ for the dislocation and friction glide resistances, respectively.

In the case of DSA, the concentration *in relations (2–4) is no longer the nominal one, but it is the local concentration in the mechanically relevant sites. The concentration dependencies of dislocation and friction parts of glide resistance were treated in the previous paper [2]. One can write*

$$\sigma_d + \sigma_f = \sigma_0 \left\{ kT \ln\left(\frac{\dot{\varepsilon}}{\nu_0}\right), \rho, c_0 \right\} + f_0 \left\{ kT \ln\left(\frac{\dot{\varepsilon}}{\nu_0}\right), \rho \right\} \frac{\Delta c}{\Delta c_M}, \tag{5}$$

where $\Delta c = c - c_0$ is the local concentration increment due to DSA, c_0 *is the nominal concentration,* Δc_M is the maximum (buffer) concentration increment,

σ_0 is the flow resistance without DSA. The structure variations of $\ln\Omega, \ln\omega$ *are omitted in (5) for simplicity. The diffusion kinetics is usually expressed by*

$$\Delta c = c - c_0 = \Delta c_M \left[1 - \exp\{-(t_a/t_0)^p\}\right], \tag{6}$$

where t_a is the diffusion ageing time. The exponent p is typically 1/3 or 2/3 [3, 4]. The diffusion relaxation time t_0 depends on the binding energy between a dislocation and solute atom, on solute concentration, and is inversely proportional to the diffusion coefficient of solutes [5].

A lot of mechanical effects due to DSA, including the critical conditions for the occurrence of PLC, has been at least qualitatively explained under identifying of ageing and waiting times:

$$t_a = t_w = \frac{\Omega}{\dot{\varepsilon}}. \tag{7}$$

Thus, Δc increases with decreasing strain rate. The strain rate sensitivity of the flow resistance (5) decreases with respect to the always positive sensitivity of a non-aged stress σ_0. However, the deformation regime with well developed PLC-bands cannot be explained in the frame of the above quasistationary treatment of DSA, unless certain *ad hoc* assumptions are made. In the next, we suggest a consistent model that yields satisfactory results also for the regime of instability propagation.

3. A concept of the PLC-instability propagation

The flow stress and strain rate are determined by such microscopic quantities as pinning force, length of pinned dislocation segments, elementary strain, and waiting time. Because these quantities concern a great number of pinning points, they obviously have a statistical character, see e.g. [6]. Variation of one of the external variables $\sigma, \dot{\varepsilon}$ transfers to the variation of the second variable through time dependent distribution functions of the fundamental microscopic quantities [7]. The use of the effective values of these quantities (instead of suitable averaging procedures) in the right sides of relations (1–4) may be rightful only if certain quasistationarity and area homogeneity of dislocation displacement are reached. In the following we refer to such a plastic regime as quieted.

In the absence of DSA the quieted flow is reached so quickly that it is possible, practically always, to use the formulation (1–4). This means, among others, that sudden changes in strain rate are followed by sudden flow stress changes.

In the case of DSA, an additional diffusion mechanism modifies the distribution of microscopic quantities: the strength of pinning points as well as solute concentration along the waiting segment pairs depend on the actual waiting times at those points. This leads to a total change of the microscopic character of slip and to a new response between external variables $\sigma, \dot{\varepsilon}$. The quieting of plastic flow

from a general initial state (as prepared, e.g. by a strain rate jump) requires that all segment pairs (also with the longest waiting time) overcome the initial obstacles and that ageing at new obstacles starts in fully quieted field of solutes. The time necessary for the establishment of the quieted state may be rather long, as it is demonstrated by the flow stress transients after strain rate jumps in tensile tests [8].

We believe that models of DSA operating with effective relations (6) and (7), e.g. [9], apply to quieted plastic regimes only. However, this is not the case of PLC. Hence, in agreement with (6) and (7), we assume the quieted plastic state in DSA regime is uniquely parametrized by $\dot{\varepsilon}$, i. e. it would be reached in a material element if a constant rate $\dot{\varepsilon}$ could be held there for a sufficiently long time.

For non-quasistationary regimes we assume that the applied stress equals a certain transient flow resistance that can be divided, similar to (5), into a fast constitutive part $\sigma_0(\dot{\varepsilon})$ with normal kinetics and a non-constitutive part σ_a:

$$\sigma = \sigma_0(\dot{\varepsilon}) + \sigma_a. \tag{8}$$

Quieting (at constant $\dot{\varepsilon}$) means that quantity (8) approaches the quieted flow stress $\sigma_F(\dot{\varepsilon})$, modelled, e.g. by (5–7). The transient behaviour is given by approaching of σ_a to the constitutive quantity $\sigma_F(\dot{\varepsilon}) - \sigma_0(\dot{\varepsilon})$. We use the simplest relaxation kinetics

$$t_1 \frac{\partial \sigma_a}{\partial t} + \sigma_a = \sigma_F(\dot{\varepsilon}) - \sigma_0(\dot{\varepsilon}), \tag{9}$$

where the relaxation time t_1 corresponds to the accommodation of the distributions of fundamental microscopic quantities. Eq. (9) is easily applicable also for non-constant $\dot{\varepsilon}$. (Note that generalization of the relaxation behaviour is possible, e.g. by dividing of σ_a into several components governed by kinetics (9) but with various relaxation times). Combining (8) and (9) we obtain

$$t_1 \dot{\sigma} + \sigma = t_1 \dot{\sigma}_0 + \sigma_F(\dot{\varepsilon}). \tag{10}$$

This equation may be considered as a generalization of the constitutive ("quieted") relation

$$\sigma = \sigma_F(\dot{\varepsilon}) \tag{11}$$

also for transient regimes. Eq. (10) applies for strain rate jumps, as well because the corresponding stress changes are given by the fast component: $\Delta\sigma = \Delta\sigma_0$.

PLC-deformation inhomogeneities (bands) are of the width of 0.1–1 mm. This is the evidence for non-local interactions which influence the slip resistance in that range. Many such mechanisms of pure material origin are known [10]. The effect of

incompatibilities, which arise during deformation of polycrystals [11], was estimated as the "strongest" among them. The corresponding internal stress $-C\varepsilon''$ may be regarded as a long range component of the local flow resistance

$$\sigma_{F,L}(x, \dot{\varepsilon}) = \sigma_F(\dot{\varepsilon}) + C\varepsilon'', \tag{12}$$

where $C = \frac{1}{4}\mu d^2$, μ is the shear modulus, d is the grain size, primes denote the differentiation with respect to x. Another kind of nonlocalities is due to the boundary conditions for stress and strain rate tensors. The free surface of a specimen is distorted at nonuniform deformation and one-component description is no more possible. For small distortions of this kind the following correction may be used

$$F_t\sigma_T = \sigma_F^*; \tag{13}$$

$$\sigma_T = \frac{P}{S_0(x)}\mathrm{e}^{\varepsilon(x)}, \tag{14}$$

where σ_T is the true stress, P is the load, $S_0(x)$ is the initial cross section (for $\varepsilon = 0$), x is the coordinate along the (deformed) specimen, F_t is the Bridgman factor

$$F_t = 1 + \frac{\beta}{8\pi}S_0 \cdot \mathrm{e}^{-\varepsilon}(\varepsilon - \ln S_0)''. \tag{15}$$

For $\beta = 1$ the formula (15) approximates the traditional one $1/F_t = (1 + 2R/y)$ $\ln(1 + y/2R)$ with the local specimen radius y and local curvature radius R of the axial cut, which has been derived [12] for a cylindrical sample and at the plane of symmetry of the neck, assuming the uniaxial flow stress σ_F^* is a quite uniform quantity. To apply the Bridgman correction to locally variant flow resistances with strain hardening, strain rate sensitivity, and at the general cross section, as well, we identify the right side of (13) with the left side of (12). We expect that the possible deviations may be involved by the factor β, assuming the gradient terms are relatively small.

Application of the basic relaxation equation (10) to non-local stress quantities evokes some questions: Should be the relaxation made for purely constitutive limit or for the local flow resistance (12), or at the "true level" (i.e. including the Bridgman correction), again without or with the incompatibility stresses? Assuming the non-quieted terms $t_1\partial/\partial t$ and the nonlocal terms with ε'' are relatively small with respect to the constitutive quantities, we neglect terms with $t_1\varepsilon''$. Then all the above mentioned variants give the same result

$$t_1\dot{\sigma}_T + F_t\sigma_T - C\varepsilon'' = t_1\dot{\sigma}_0 + \sigma_F. \tag{16}$$

In frame of the last used approximation, the only constitutive part of $\dot{\sigma}_0$ is relevant in (16), too. The relation (16) represents the equation for propagation of PLC--instabilities.

4. Numerical checks of propagation equation

To check numerically Eq. (16), we bear upon the tensile tests of an Al-3Mg commercial alloy. The samples of gauge length 50 mm and cross section 1.5×6 mm were deformed on INSTRON 1195 machine at room temperature and the crosshead speed $U = 1.67 \times 10^{-3}$ mm/s, i.e. at applied rate $\dot{\varepsilon}_a = 3.3 \times 10^{-5}$ s^{-1}. The machine condition reads

$$Ut + x_0 = kP + \int_0^{l_0} \varepsilon(x,t)\mathrm{d}x, \tag{17}$$

where $k = 1.7 \times 10^{-4}$ mm/N is the compliance of our device. Using (14) and (17) the true stress and true stress rate are expressed by integrals of strain and strain rate along the specimen. To express analytically the quieted flow stress, we combine our experimental results with recalculated data reported for Al-5Mg and strain of 8% in [13]. Our expression for $\sigma_F(\dot{\varepsilon}, \varepsilon)$ is not reproduced here, but its fit to empirical data can be seen in Fig. 1. The high rate branch of quieted flow stress, if expanded to all strain rates, serves as immediate (fast) flow resistance:

$$\sigma_0(\dot{\varepsilon}, \varepsilon) = h(\varepsilon - \varepsilon_c) + M \operatorname{arcsinh}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right); \tag{18}$$

here $h = 1013$ MPa for $\varepsilon_c = 0.08$. The fitted value $M = 6.28$ MPa corresponds practically to the familiarly defined immediate sensitivity $\partial\sigma_0/\partial \ln \dot{\varepsilon}$; the arcsinh function was used only to avoid difficulties for $\dot{\varepsilon} \to 0$. A typical experimental load signal is shown in Fig. 2.

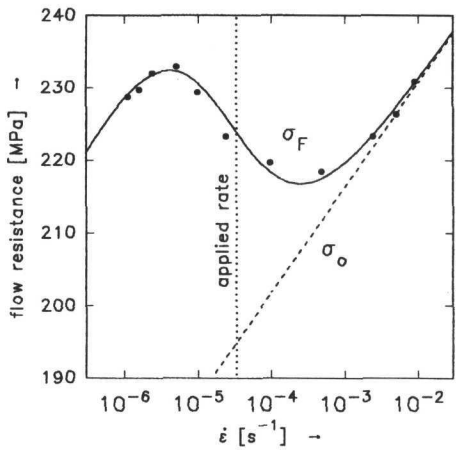

Fig. 1. Quieted and immediate slip resistances.

With (18) and (17) the relation (16) becomes an integro-differential partial equation of second order in time and coordinate for $\varepsilon(x,t)$. It appears that inhomogeneity propagation is rather sensitive to the factors at $\ddot{\varepsilon}$ and $\varepsilon'' : t_1 M/\sqrt{\ddot{\varepsilon}^2 + \dot{\varepsilon}_0^2}$ and $\beta P/8\pi - C$, respectively. In our case, $P/8\pi$ is of about 80 N and $\mu = 26.8$ GPa, grain size $d = 18$ μm which result in $C = 2.2$ N. So, the incompatibility contribution to the non-local interactions seems to be small with respect to the

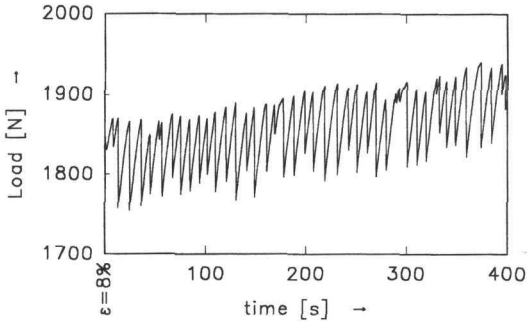

Fig. 2. Load signal in tensile test, Al-3Mg.

Bridgman one. (Note that in these considerations both β as well as "coherence distance" d remain rather uncertain.) In the next, we formally set $C = 0$ and try out the value of β. As to the coordinate profile of the initial cross section, we allow an exponential law in order to reach $(\ln S_0)'' = 0$. The boundary values are $S_0 = 8.9$ and 9 mm^2 at the left and right ends of the specimen, respectively; this corresponds to the normal machining.

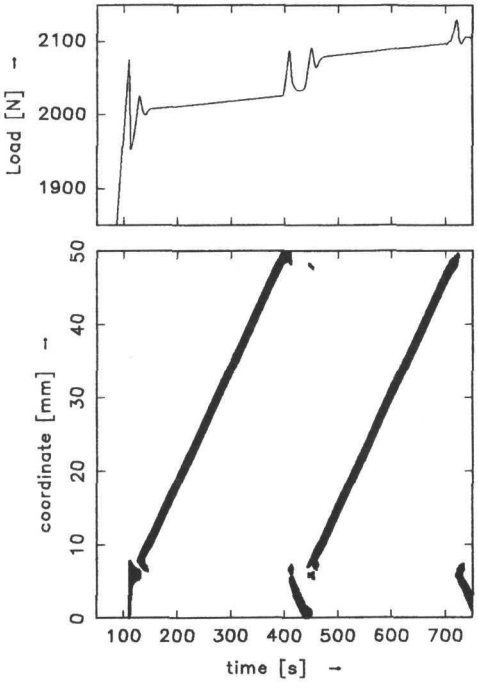

Fig. 3. Calculated instability propagation, $t_1 = 0.1$, $\beta = 1$.

The examples of calculations are shown at Figs. 3, 4, 5 for various combinations of β and t_1. The time-coordinate diagrams were obtained as "isohypses" of the $\dot{\varepsilon}(t, x)$-surface at the level of applied rate. The black and white areas correspond to strain rates above and below the applied one, respectively. In spite of great complexity of our propagation equation and of wide variety of its solutions, we can remark a tendency governed by the ratio β/t_1 (i.e. relative "strength" of non-local interactions with respect to relaxation time): at greater ratios, the deformation spreads in compact smooth waves, named empirically as type A of PLC (Fig. 3). Decreasing the ratio,

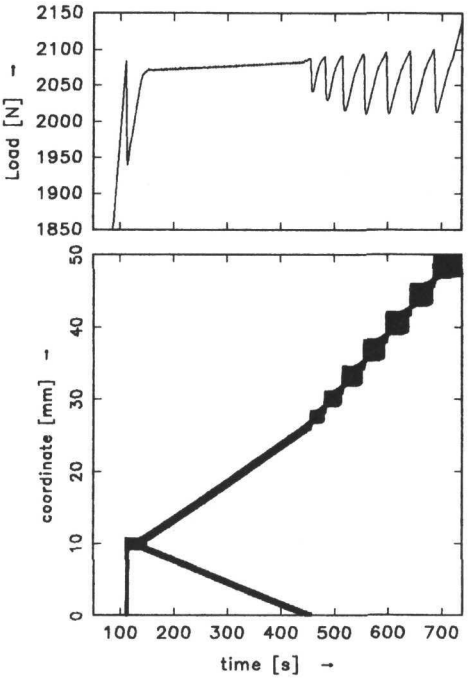

Fig. 4. Calculated instability propagation, $t_1 = 0.2$, $\beta = 0.05$.

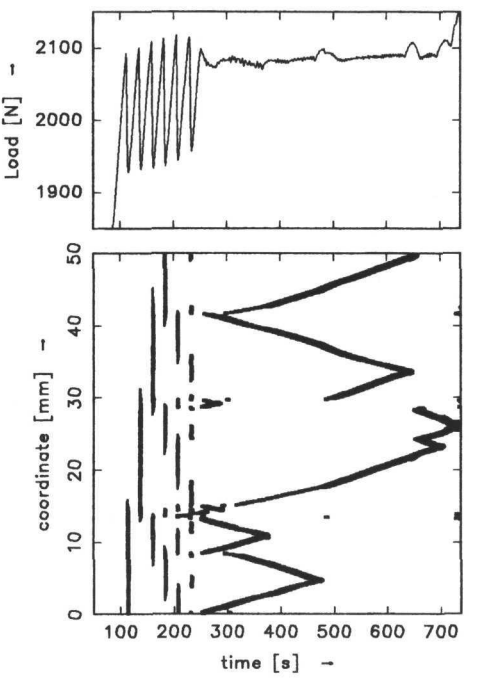

Fig. 5. Calculated instability propagation, $t_1 = 0.5$, $\beta = 0.1$.

both propagation rate and coherency decrease, too. Either nucleation of opposite waves or hopping deformation bands appear, known as B-type of PLC (Fig. 4). Still lower values of β/t_1 can lead to full loss of time-space coherency and to individual deformation bursts chaotically distributed in space – see the first part in Fig. 5. This regime is accompanied with pronounced load drops and we suggest to identify it with the C-type of PLC. The following regime in Fig. 5 is represented by multiple waves of relatively slow propagation rate and of "fine hopping structure". A high sensitivity of the propagation mode to the strain and strain rate profiles, as they are build up during deformation, is manifested by transitions in Figs. 4, 5.

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