EFFECTIVE YOUNG'S MODULUS OF QUASILAYERED SPECIMENS: COMPARISON OF EXPRESSIONS CORRESPONDING TO TENSILE TESTS AND DYNAMICAL RESONANT METHODS

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General expressions for effective Young's modulus of quasilayered specimens were derived for model situations corresponding to statical tensile tests and dynamical resonant tests. These expressions enable us to evaluate the effective-modulus values which would be measured if the quasilayered specimens were undergone tensile tests and dynamical resonant tests. General expressions for effective Young's modulus based on sample responses (elongation, vibration) occuring in the plane of quasilayers, are equal for the both testing-method sets. On the other hand, general expressions as well as most of particular values for the effective Young's modulus, based on the sample responses perpendicular to quasilayers, differ from each other and from those for "in plane" modulus. So, for a design use, the modulus value determined by means of the method compatible with the application intended should be chosen.

EFEKTÍVNY YOUNGOV MODUL PRUŽNOSTI KVÁZIVRSTEVNATÝCH VZORIEK: POROVNANIE VÝRAZOV ZODPOVEDAJÚCICH ŤAHOVÝM SKÚŠKAM A DYNAMICKÝM REZONANČNÝM METÓDAM

Odvodili sme všeobecné výrazy pre efektívny Youngov modul pružnosti kvázivrstevnatých vzoriek pre modelové situácie zodpovedajúce statickým ťahovým skúškam a dynamickým rezonančným skúškam. Tieto výrazy nám umožňujú vypočítať hodnoty efektívneho modulu, ktoré by sme namerali, ak by sme podrobili kvázivrstevnaté vzorky ťahovým skúškam a dynamickým rezonančným skúškam. Všeobecné výrazy pre efektívny Youngov modul, založené na odozvách vzorky (predĺženie, vibrácia) v rovine kvázivrstiev, sú rovnaké pre oba súbory skúšobných metód. Na druhej strane, všeobecné výrazy, ako aj väčšina konkrétnych hodnôt pre efektívny Youngov modul, založené na odozvách vzorky kolmých na kvázivrstvy, sa navzájom odlišujú a odlišujú sa aj od výrazov a hodnôt pre

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moduly „v rovine". Pri projektovaní je preto potrebné vybrať hodnotu modulu určenú pomocou metódy kompatibilnej s plánovaným použitím výrobku.

Key words: effective Young's modulus, quasilayered specimens, tensile test, dynamic resonant method

1. Introduction

Moduli of elasticity represent one of the important mechanical characteristics of materials. Acquaintance with moduli values is necessary, for example, for designing various structural parts as the response of a particular part to the action of external forces is largely determined by the elasticity-modulus values [1]. On the other hand, various sample responses (elongation, bending, vibration, etc.) to the given external stimuli can be used for determining the elasticity moduli and serve as a physical basis for a variety of modulus-determining methods.

In the case of homogeneous samples, the elasticity-modulus values obtained by means of different testing methods are equal (except for, of course, the measuring errors), regardless of the sample geometrical shape and the method being used. The values determined in this case are equal to the modulus of elasticity of material the samples are made of.

Recently, intentionally macroscopically heterogeneous materials and parts are increasingly used (composite materials, powder-metallurgy parts with specially modified surface, etc.). For working reasons, it is necessary to evaluate the response of such parts to various external stimuli, too. Therefore, a number of effective moduli of elasticity is introduced [2–4]. Effective modulus is a modulus of material of a hypothetical homogeneous component of the same size and shape as the tested heterogeneous one, which provides the same response to a particular stimulus as the real heterogeneous component. The effective modulus, understood in the above mentioned sense, is a function of moduli of particular materials constituting the component as well as it depends also on the distribution of particular materials within the component and on a geometrical shape of the component as a whole.

Therefore, when different modulus-determining procedures are applied to a heterogeneous sample, different values of the effective modulus are obtained. Thus, when we are dealing with heterogeneous materials or components, the concept of effective modulus as well as values provided by particular tests should be used carefully. One has to keep in mind which procedure (test) was used for determining a particular value of effective modulus and for which kind of practical or research situations this value can be used.

In the paper presented the effective-modulus differences are illustrated by expressions theoretically derived for effective Young's moduli of a quasilayered bar. These expressions enable us to evaluate effective Young's modulus values which would be provided by two different testing methods: statical tensile test and a dynamical resonant method. The general expressions for effective Young's moduli are presented as well as some particular Young's moduli for particular situations (bars with a special distribution of (quasi) layers) are evaluated.

The results confirm the statement that for a heterogeneous bar different testing methods can provide different effective-modulus values. Thus, some care is needed to choose the correct modulus value for a given component application, i.e., it is necessary to choose the modulus value determined by means of a testing procedure compatible with the application intended for a given part.

2. General theoretical expressions for effective Young's moduli of quasilayered bars

To obtain expressions for effective Young's modulus in a theoretical way, a rectangular bar of height H, width W, and length L is considered as a theoretical model of the sample undergoing particular tests. The x-axis of a co-ordinate frame is oriented along the height of the bar (Fig. 1).

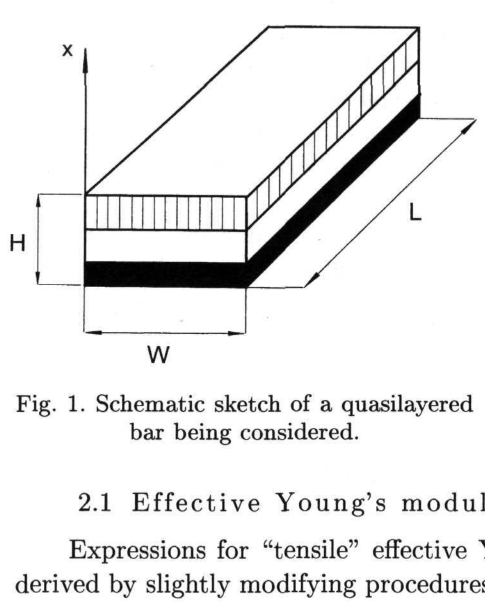


Fig. 1. Schematic sketch of a quasilayered bar being considered.

Properties of material of the bar (material Young's modulus E, mass density ρ) are assumed as varying (continuously or step-likely) only along the height of the bar. So, the Young's modulus $E(x)$ and density $\rho(x)$ are functions of the co-ordinate x only. Such a bar is called a quasilayered one.

2.1 Effective Young's moduli determined by tensile tests

Expressions for "tensile" effective Young's moduli for our model bar can be derived by slightly modifying procedures used for deriving the effective moduli for a slab model of unidirectional composites [2, 3, 4].

Transverse modulus

A tensile stress σ is applied in the direction of the bar height. In this case, all bar cross sections parallel to WL-plane experience equal stress σ. Elongation $\mathrm{d}l(x)$ of a thin layer of original thickness $\mathrm{d}x$, parallel to WL-plane and located at height x, can be expressed as $\mathrm{d}l(x) = \sigma \mathrm{d}x/E(x)$. The total elongation of the bar in the direction of height is obtained by integration of $\mathrm{d}l(x)$ over the height of the bar.

In this case, the effective bar modulus in tension $E_{\perp}^{\mathrm{t}}$ is defined by relation:

$$\sigma = E_{\perp}^{\mathrm{t}} \frac{\int \mathrm{d}l(x)}{H}.$$

Substituting proper relations and performing simple mathematical operations, we have for $E_{\perp}^{\mathrm{t}}$ expression as follows:

$$E_{\perp}^{\mathrm{t}} = \left(\frac{1}{H} \int_0^H \frac{\mathrm{d}x}{E(x)} \right)^{-1} . \tag{1}$$

For actual layered bar $E_{\perp}^{\mathrm{t}}$ obtains the form known as "Inverse Rule of Mixtures".

Longitudinal modulus

If the bar is strained in the direction of length L (or width W), the strains ε in all quasilayers (bar cross sections parallel to WL-plane) are equal and are the same as the bar strain. The force $\mathrm{d}f(x)$ borne by a thin layer of thickness $\mathrm{d}x$, parallel to WL-plane and located at the height x, can be expressed as $\mathrm{d}f(x) = \varepsilon E(x) W \mathrm{d}x$. The total force borne by the strained bar is obtained by integration of $\mathrm{d}f(x)$ over the bar height.

In this case, the effective bar modulus in tension $E_{\parallel}^{\mathrm{t}}$ is defined by relation

$$\frac{\int \mathrm{d}f(x)}{WH} = E_{\parallel}^{\mathrm{t}} \varepsilon.$$

Substituting proper relations and performing simple mathematical operations, we have for $E_{\parallel}^{\mathrm{t}}$ the following expression:

$$E_{\parallel}^{\mathrm{t}} = \frac{1}{H} \int_0^H E(x) \mathrm{d}x. \tag{2}$$

For an actual layered bar $E_{\parallel}^{\mathrm{t}}$ obtains the form known as "Rule or Law of Mixtures". Rule of mixtures is often used for determining the effective modulus of multiphase materials.

2.2 Effective Young's moduli determined by a dynamical resonant method

In this paragraph we concentrate on the effective Young's modulus determined by means of a dynamical resonant method [5], using flexural vibration of a bar being tested. When the method is applied to our model quasilayered bar (Fig. 1), the effective Young's modulus values are evaluated by means of the following expressions [5]:

– for vibration in the HL-plane:

$$E_{\perp}^{\mathrm{v}} = 0.94642 \rho_{\mathrm{av}} L^4 \frac{f_{\perp}^2}{H^2}, \tag{3a}$$

– for vibration in the WL-plane:

$$E_{\parallel}^{\mathrm{v}} = 0.94642\rho_{\mathrm{av}}L^4\frac{f_{\parallel}^2}{W^2}. \tag{3b}$$

ρ_{av} is the averaged bar mass density. $f_{\perp}$ and $f_{\parallel}$ are experimentally measured fundamental frequencies of our quasilayered bar, undergoing transverse vibration with free ends in the HL-plane and WL-plane, respectively.

To evaluate the moduli $E_{\perp}^{\mathrm{v}}, E_{\parallel}^{\mathrm{v}}$ in a theoretical way, we have to calculate the fundamental frequencies $f_{\perp}, f_{\parallel}$ of our model bar as functions (functionals) of the material-property distribution within the bar.

As a transverse vibration of a quasilayered bar is considered, we have to modify the Bernoulli-Euler beam theory [6] (which provided relations (3a,b)) to allow for the quasilayered structure of the bar under consideration.

Vibration frequencies are obtained by solving the equation of motion for a bar neutral fibre [6]. The equation of motion can be derived by means of the Hamilton's principle of minimal action. Lagrange's function, occurring in the expression for action, consists of the kinetic-energy part and the potential (elastic)-energy part.

Required elastic energy is determined by means of the strain and stress tensor fields within a bent quasilayered bar. The geometry of deformation of material fibres and planar bar cross sections in a bent quasilayered bar is similar to that in a homogeneous bent bar. That is, the elongation (contraction) of a material fibre in a given point of cross section of a bent bar increases linearly with increasing distance of a point considered from the cross-section neutral axis. The cross-section neutral axis is perpendicular to the bending plane, and its position within the cross section is determined by conditions $\iint_S E(h)h\mathrm{d}h\mathrm{d}w = 0$ (bending in the HL-plane) or $\iint_S E(h)w\mathrm{d}h\mathrm{d}w = 0$ (bending in the WL-plane). Integration runs over the cross section S. Quantities h and w are distances of a given area-element $\mathrm{d}h\mathrm{d}w$ from the cross-section neutral axes for bending in the HL-plane and WL-plane, respectively.

Taking into account above-mentioned character of deformation as well as a heterogeneous distribution of material Young's modulus value along the cross section, the expression for the elastic energy can be constructed. The resultant elastic energy of a bent quasilayered bar, if expressed in terms of the neutral-fiber curvature, differs from the elastic energy of a bent homogeneous bar only in a prefactor, where expressions $\iint_S E(h)h^2\mathrm{d}h\mathrm{d}w$ (bending in the HL-plane) or $\iint_S E(h)w^2\mathrm{d}h\mathrm{d}w$ (bending in the WL-plane) play the role of the flexural rigidity of the quasilayered bar.

The kinetic energy of a quasilayered bar, if expressed in terms of the velocity of the neutral-fiber transverse motion, differs from that of a homogeneous bar only in the prefactor $\iint_S \rho(h)\mathrm{d}h\mathrm{d}w$ instead of $HW\rho_0$.

These differences lead to analogous changes of corresponding quantities in the equation of motion and relation for natural frequencies of a homogeneous bar when rederived for a quasilayered bar.

Transforming relevant quantities from a hw-frame to the co-ordinate frame of Fig. 1 and substituting calculated frequencies $f_\perp, f_\parallel$ into Eqs. (3), the following expressions for "transverse vibration" effective Young's moduli are obtained:

– for vibration in the HL-plane, i.e. perpendicular to quasilayers

$$E_\perp^{\rm v} = \frac{12}{H^3}\left[\int_0^H E(x)x^2\mathrm{d}x - \frac{\left(\int_0^H E(x)x\mathrm{d}x\right)^2}{\int_0^H E(x)\mathrm{d}x}\right], \tag{4}$$

– for vibration in the WL-plane, i.e. parallel to quasilayers

$$E_\parallel^{\rm v} = \frac{1}{H}\int_0^H E(x)\mathrm{d}x. \tag{5}$$

Expressions for $E_\parallel^{\rm t}$ and $E_\parallel^{\rm v}$ are equal. Expressions for $E_\perp^{\rm t}$ and $E_\perp^{\rm v}$ are different from $E_\parallel^{\rm t}, E_\parallel^{\rm v}$ and from each other.

3. Some particular examples of effective Young's moduli

To illustrate the differences among effective Young's moduli determined by different testing methods, the general results of the above chapter are employed for evaluating the effective moduli for bars with some special distributions of material Young's modulus along the bar cross section.

As an example of a bar with a step-like distribution of material Young's modulus (actual layered bar), a bar consisting of three layers is considered. Two outer layers are of thickness $H_{\rm S1} = \nu_{\rm S1}H$ and $H_{\rm S2} = \nu_{\rm S2}H$ and material Young's moduli within these layers are $E_{\rm S1}$ and $E_{\rm S2}$, respectively. The central layer is of thickness $H_{\rm B} = \nu_{\rm B}H$ and with a material Young's modulus $E_{\rm B}$ (Fig. 2). ν_i's are the volume fractions occupied by particular layers, i.e., the ratios of the volume of the i-th layer to the volume of the whole bar.

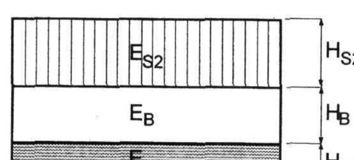


Fig. 2. Schematic sketch of the cross section of an arbitrary three-layer bar.

For such three-layer bar the expressions for effective Young's moduli are as follows:

$$E_{\parallel}^{\mathrm{v}} = E_{\parallel}^{\mathrm{t}} = \nu_{\mathrm{S1}} E_{\mathrm{S1}} + \nu_{\mathrm{B}} E_{\mathrm{B}} + \nu_{\mathrm{S2}} E_{\mathrm{S2}}, \qquad \text{rule of mixtures} \tag{6a}$$

$$E_{\perp}^{\mathrm{t}} = \frac{E_{\mathrm{S1}} E_{\mathrm{B}} E_{\mathrm{S2}}}{\nu_{\mathrm{S1}} E_{\mathrm{B}} E_{\mathrm{S2}} + \nu_{\mathrm{B}} E_{\mathrm{S1}} E_{\mathrm{S2}} + \nu_{\mathrm{S2}} E_{\mathrm{S1}} E_{\mathrm{B}}}, \qquad \text{inverse rule of mixtures} \tag{6b}$$

$$E_{\perp}^{\mathrm{v}} = \frac{A + B_{\mathrm{S1,B}} + B_{\mathrm{S1,S2}} + B_{\mathrm{B,S2}} + C}{\nu_{\mathrm{S1}} E_{\mathrm{S1}} + \nu_{\mathrm{B}} E_{\mathrm{B}} + \nu_{\mathrm{S2}} E_{\mathrm{S2}}}, \tag{6c}$$

$$A \equiv \nu_{\mathrm{S1}}^4 E_{\mathrm{S1}}^2 + \nu_{\mathrm{B}}^4 E_{\mathrm{B}}^2 + \nu_{\mathrm{S2}}^4 E_{\mathrm{S2}}^2,$$
$$B_{\mathrm{i,j}} \equiv (4\nu_{\mathrm{i}}^2 + 6\nu_{\mathrm{i}}\nu_{\mathrm{j}} + 4\nu_{\mathrm{j}}^2)\nu_{\mathrm{i}}\nu_{\mathrm{j}} E_{\mathrm{i}} E_{\mathrm{j}},$$
$$C \equiv 12 \nu_{\mathrm{S1}} \nu_{\mathrm{B}} \nu_{S2} E_{\mathrm{S1}} E_{\mathrm{S2}},$$
$$\nu_{\mathrm{S1}} + \nu_{\mathrm{B}} + \nu_{\mathrm{S2}} = 1.$$

For a symmetric three-layer bar ($E_{\mathrm{S1}} = E_{\mathrm{S2}} = E_{\mathrm{S}}$; $\nu_{\mathrm{S1}} = \nu_{\mathrm{S2}} = 0.5\nu_{\mathrm{S}}$) expressions (6) transform to the form:

$$E_{\parallel}^{\mathrm{v}} = E_{\parallel}^{\mathrm{t}} = \nu_{\mathrm{S}} E_{\mathrm{S}} + \nu_{\mathrm{B}} E_{\mathrm{B}}, \tag{7a}$$

$$E_{\perp}^{\mathrm{t}} = \frac{E_{\mathrm{S}} E_{\mathrm{B}}}{\nu_{\mathrm{S}} E_{\mathrm{B}} + \nu_{\mathrm{B}} E_{\mathrm{S}}}, \tag{7b}$$

$$E_{\perp}^{\mathrm{v}} = E_{\mathrm{S}} + (E_{\mathrm{B}} - E_{\mathrm{S}})\nu_{\mathrm{B}}^3. \tag{7c}$$

In Fig. 3, the effective Young's moduli for a symmetric three-layer bar are presented as functions of the outer-layers volume fraction ν_{S} for the situation $E_{\mathrm{S}}/E_{\mathrm{B}} = 0.7$. The differences in values of particular moduli are apparent.

For a two-layer bar ($\nu_{\mathrm{B}} = 0$) expressions (6) reduce to the form:

$$E_{\parallel}^{\mathrm{v}} = E_{\parallel}^{\mathrm{t}} = \nu_{\mathrm{S1}} E_{\mathrm{S1}} + \nu_{\mathrm{S2}} E_{\mathrm{S2}}, \tag{8a}$$

$$E_{\perp}^{\mathrm{t}} = \frac{E_{\mathrm{S1}} E_{\mathrm{S2}}}{\nu_{\mathrm{S1}} E_{\mathrm{S2}} + \nu_{\mathrm{S2}} E_{\mathrm{S1}}}, \tag{8b}$$

$$E_{\perp}^{\mathrm{v}} = \frac{\nu_{\mathrm{S1}}^4 E_{\mathrm{S1}}^2 + 2(2 - \nu_{\mathrm{S1}}\nu_{\mathrm{S2}})\nu_{\mathrm{S1}}\nu_{\mathrm{S2}} E_{\mathrm{S1}} E_{\mathrm{S2}} + \nu_{\mathrm{S2}}^4 E_{\mathrm{S2}}^2}{\nu_{\mathrm{S1}} E_{\mathrm{S1}} + \nu_{\mathrm{S2}} E_{\mathrm{S2}}}. \tag{8c}$$

In Fig. 4, the effective Young's moduli for a two-layer bar are presented as functions of the volume fraction ν_{S1} of one of the layers for the situation with $E_{\mathrm{S1}}/E_{\mathrm{S2}} = 0.7$. Also in this case the differences in modulus values are apparent.

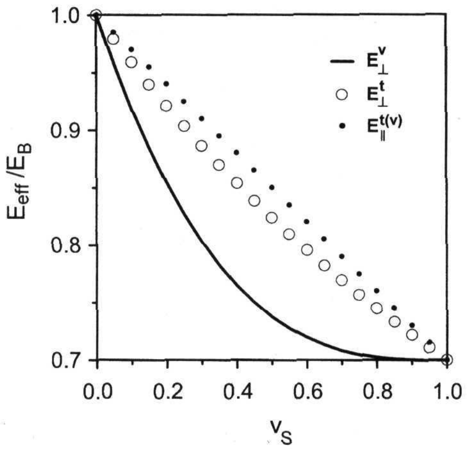


Fig. 3. Normalized effective Young's moduli vs. the outer-layers volume fraction ν_S. The data presented were calculated for a symmetric three-layer bar with the outer layer-to-central layer Young's moduli ratio E_S/E_B equal to 0.7.

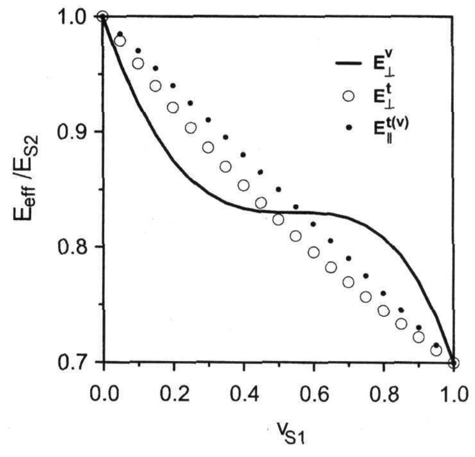
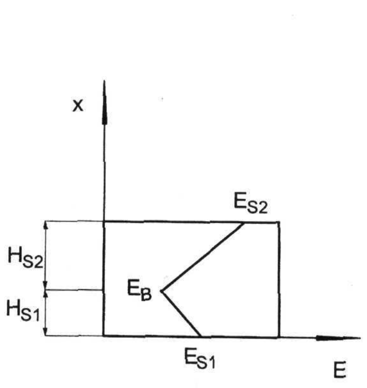


Fig. 4. Normalized effective Young's moduli vs. the volume fraction ν_{S1} of one of the layers. The data presented were calculated for a two-layer bar with the layers Young's moduli ratio E_{S1}/E_{S2} equal to 0.7.

Fig. 5. Schematic sketch of the material Young's modulus distribution along the height of the cross section of a bar with properties varying linearly with a distance from the surface.



As examples of bars with a continuous distribution of the material Young's modulus, a bar consisting of two segments with properties varying linearly with a distance from surfaces (Fig. 5) and a bar with a symmetric quadratic modulus distribution are considered.

In a two-segment bar, the material Young's modulus changes linearly from the value E_{S1} on one surface to the value E_B on some plane inside the bar at a distance $H_{S1} = h_{S1}H$ from the surface considered, and then it changes linearly, in general with a different slope, from the value E_B to the value E_{S2} on the opposite surface.

For a bar with such material-modulus distribution, effective Young's moduli are as follows:

$$E_{\parallel}^{v} = E_{\parallel}^{t} = h_{S1}\frac{E_{S1}+E_B}{2} + h_{S2}\frac{E_{S2}+E_B}{2}, \tag{9a}$$

$$E_{\perp}^{t} = \left[h_{S1}\frac{\ln\dfrac{E_{S1}}{E_B}}{E_{S1}-E_B} + h_{S2}\frac{\ln\dfrac{E_{S2}}{E_B}}{E_{S2}-E_B}\right]^{-1}, \tag{9b}$$

$$E_{\perp}^{v} = \frac{K_{S1}+N_{S1,S2}+M+N_{S2,S1}+K_{S2}}{3[h_{S1}(E_{S1}+E_B)+h_{S2}(E_{S2}+E_B)]}, \tag{9c}$$

$$K_i \equiv h_i^4(E_i^2+4E_iE_B+E_B^2),$$
$$N_{i,j} \equiv 3h_i^3h_j(E_B+3E_i)(E_B+E_j),$$
$$M \equiv 4h_{S1}^2h_{S2}^2(E_B+2E_{S1})(E_B+2E_{S2}),$$
$$h_{S1}+h_{S2}=1.$$

For a symmetric two-segment bar ($E_{S1}=E_{S2}=E_S$, $h_{S1}=h_{S2}=0.5$) with linearly varying material properties the expressions (9) reduce to the forms:

$$E_{\parallel}^{v} = E_{\parallel}^{t} = \frac{E_S+E_B}{2}, \tag{10a}$$

$$E_{\perp}^{t} = \frac{E_S-E_B}{\ln\dfrac{E_S}{E_B}}, \tag{10b}$$

$$E_{\perp}^{v} = \frac{E_B+3E_S}{4}. \tag{10c}$$

In Fig. 6, normalized effective Young's moduli for a symmetric two-segment bar are presented as functions of the ratio E_S/E_B. In this case, the values of modulus $E_{\perp}^{t}$ are almost equal to values of moduli $E_{\parallel}^{v}=E_{\parallel}^{t}$ for all values of the ratio E_S/E_B from the interval considered. But the values of $E_{\perp}^{v}$ still differ from the others.

For a bar consisting only of one segment with linearly changing material Young's modulus ($h_{S1}=0, E_B=E_{S1}$), expressions (9) reduce to the forms:

$$E_{\parallel}^{v} = E_{\parallel}^{t} = \frac{E_{S1}+E_{S2}}{2}, \tag{11a}$$

$$E_{\perp}^{t} = \frac{E_{S2}-E_{S1}}{\ln\dfrac{E_{S2}}{E_{S1}}}, \tag{11b}$$

$$E_{\perp}^{\mathrm{v}} = \frac{E_{\mathrm{S1}}^2 + 4E_{\mathrm{S1}}E_{\mathrm{S2}} + E_{\mathrm{S2}}^2}{3(E_{\mathrm{S1}} + E_{\mathrm{S2}})}. \tag{11c}$$

For the second "continuous" example, a bar with a symmetric quadratic material Young's modulus distribution along the bar height (with Young's modulus value E_{S} on the surfaces and E_{B} in the centre of the bar cross section), the effective Young's moduli are as follows:

$$E_{\parallel}^{\mathrm{v}} = E_{\parallel}^{\mathrm{t}} = \frac{E_{\mathrm{S}} + 2E_{\mathrm{B}}}{3}, \tag{12a}$$

$$E_{\perp}^{\mathrm{t}} = \begin{cases} \dfrac{\sqrt{E_{\mathrm{B}}(E_{\mathrm{B}} - E_{\mathrm{S}})}}{\operatorname{arg\,tgh}\sqrt{\dfrac{E_{\mathrm{B}} - E_{\mathrm{S}}}{E_{\mathrm{B}}}}} & \text{for} \quad E_{\mathrm{S}} < E_{\mathrm{B}}, \\ E_{\mathrm{B}} & \text{for} \quad E_{\mathrm{S}} = E_{\mathrm{B}}, \\ \dfrac{\sqrt{E_{\mathrm{B}}(E_{\mathrm{S}} - E_{\mathrm{B}})}}{\operatorname{arctg}\sqrt{\dfrac{E_{\mathrm{S}} - E_{\mathrm{B}}}{E_{\mathrm{B}}}}} & \text{for} \quad E_{\mathrm{S}} > E_{\mathrm{B}}, \end{cases} \tag{12b}$$

$$E_{\perp}^{\mathrm{v}} = \frac{3E_{\mathrm{S}} + 2E_{\mathrm{B}}}{5}. \tag{12c}$$

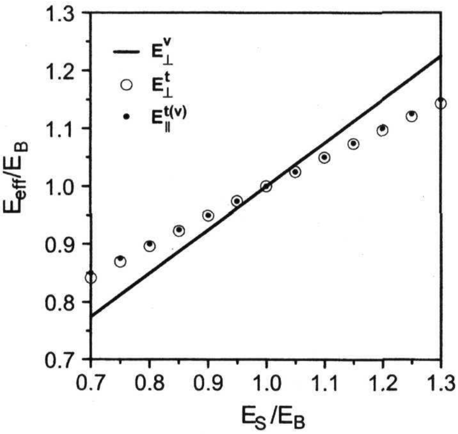


Fig. 6. Normalized effective Young's moduli vs. surface-to-centre material Young's moduli ratio $E_{\mathrm{S}}/E_{\mathrm{B}}$. The data presented were calculated for a symmetric bar with the Young's modulus continuously varying along the bar height, with values proportional to a distance from the bar centre.

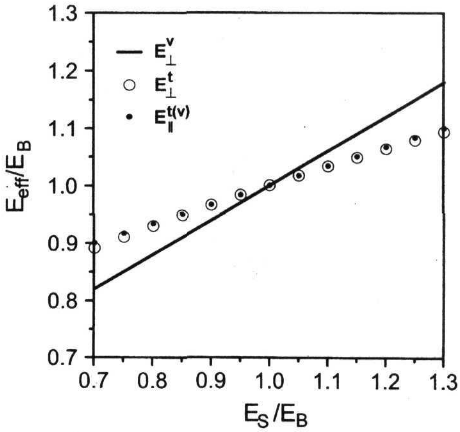


Fig. 7. Normalized effective Young's moduli vs. surface-to-centre material Young's moduli ratio $E_{\mathrm{S}}/E_{\mathrm{B}}$. The data presented were calculated for a symmetric bar with the Young's modulus continuously varying along the bar height, with values proportional to the square of a distance from the bar centre.

In Fig. 7, normalized effective Young's moduli for a symmetric „quadratic" bar are presented as functions of the ratio E_S/E_B. Also in this case only the values of modulus $E_{\perp}^{v}$ are visibly different from the others.

4. Conclusion

In the paper presented here we referred to the differences in the effective Young's modulus values which the different testing methods could provide if applied to quasilayered rectangular-bar samples. In our approach, a quasilayered bar is a heterogeneous one with a material Young's modulus varying only in one transverse direction, referred to as a bar height. The material Young's modulus distribution along the bar height can be continuous as well as a step-like one (actual layered bars).

General mathematical expressions for effective Young's moduli are derived. These expressions enable us to evaluate the effective Young's modulus values which would be determined experimentally for a quasilayered bar by means of tensile tests and dynamical resonant methods.

Two of the expressions obtained for effective Young's moduli, one derived by considering an elongation of a quasilayered bar strained in the direction parallel to quasilayers and another derived by means of a natural frequency of the same bar undergoing transverse vibration in the plane of quasilayers, are equal. But another two expressions, one derived by means of the elongation of the same bar strained in the direction perpendicular to quasilayers and another derived by means of a natural frequency of the bar vibrating perpendicularly to quasilayers, formally differ from each other and from the former.

These formal differences in particular mathematical expressions for effective Young's moduli can lead to visible differences also in numerical values evaluated by means of these expressions. It is the case of our actual three-layer and two-layer bars (Figs. 3, 4), where the curves for $E_{\perp}^{t}$ and $E_{\perp}^{v}$ differ from each other and from the curve for $E_{||}^{v} = E_{||}^{t}$. But for another bar configuration, the differences in evaluated numerical values can be very negligible. It is the case of our model bars with continuously varying properties (Figs. 6, 7), where the $E_{\perp}^{t}$ vs. E_S/E_B curve practically coincides with the $E_{||}^{t,v}$ vs. E_S/E_B curve and only the $E_{\perp}^{v}$ vs. E_S/E_B curve is visibly different.

But in general, the tensile tests and dynamical resonant method (and, of course, also another testing methods not considered in this paper) can provide different values of effective moduli for the same quasilayered (or, in general, heterogeneous) specimen. Thus, some care is needed to choose the correct effective-modulus value (determined by a correct testing method) for particular component applications.

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REFERENCES

[1] CRAIG Jr., R. R.: Mechanics of Materials. New York, Wiley 1996.
[2] GIBSON, F. R.: Principle of Composite Material Mechanics. New York, Mc Graw--Hill 1994.
[3] MATTHEWS, F. L.—RAWLINGS, R. D.: Composite Materials: Engineering and Science. London, Chapman and Hall 1994.
[4] HULL, D.—CLYNE, T. W: An Introduction to Composite Materials. Second Edition. Cambridge, Cambridge University Press 1996.
[5] SPINNER, S.—TEFFT, W. E.: Proceedings, Am. Soc. Testing Mats., *61*, 1961, p. 1221.
[6] CRAIG Jr., R. R.: Structural Dynamics - An Introduction to Computer Methods. New York, Wiley 1981.

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