THE EFFECT OF HETEROGENEOUS DISTRIBUTION OF POROSITY ON THE FLEXURAL MODULI OF SINTERED IRON BARS

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Bar shaped specimens were prepared from iron powder. The specimens possessed a quasilayered distribution of porosity with porous outer regions and more dense core. For such specimens, the effective flexural moduli determined by means of flexural vibration parallel to the quasilayers were systematically higher than those determined from vibrations perpendicular to the quasilayers. When the more porous surface regions were more or less removed by grinding, the difference between "parallel" and "perpendicular" moduli decreased or even vanished. Such effect of surface regions of less stiffness was also confirmed theoretically by calculating the relevant moduli for model bars with quasilayered distribution of porosity.

K e y w o r d s: sintered iron, heterogeneous distribution of porosity, effective flexural modulus

VPLYV NEROVNOMERNÉHO ROZDELENIA PÓROVITOSTI NA MODULY V OHYBE TYČÍ ZO SPEKANÉHO ŽELEZA

Zo železného prášku sme pripravili vzorky v tvare tyčí. Vzorky mali kvázivrstevnaté rozdelenie pórovitosti, s pórovitými okrajovými oblasťami a hustejším jadrom. Pre takéto vzorky boli efektívne moduly v ohybe určované pomocou ohybových kmitov rovnobežných s kvázivrstvami systematicky vyššie ako moduly určované pomocou kmitov kolmých na kvázivrstvy. Keď sme pórovitejšie povrchové oblasti viac alebo menej odbrúsili, rozdiel medzi hodnotami „rovnobežného" a „kolmého" modulu klesol, alebo dokonca vymizol. Takýto vplyv povrchových oblastí s nižšou tuhosťou sme potvrdili taktiež teoreticky, a

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to výpočtom relevantných modulov pre modelové tyče s kvázivrstevnatým rozdelením pórovitosti.

1. Introduction

The continuing quest for more accomplished and more efficient technical facilities is permanently forcing the materials scientists and engineers to produce more sophisticated structural parts, which are very often macroscopically heterogeneous. Parts made of composite materials, powder-metallurgy components with specially modified surface, functionally graded materials, etc., can be mentioned as examples.

For design purposes, it is necessary to evaluate the response of such structural parts to various external loading, i.e., it is desirable to know stiffness coefficients, torsional rigidity, flexural rigidity of structural parts under consideration, and so on. Sometimes, it is also useful to know effective (apparent) moduli of elasticity for parts considered, although for macroscopically heterogeneous parts the term "effective modulus" is rather vague and less informative in comparison to homogeneous parts. It is caused by the fact that, for heterogeneous parts, the overall properties are in general considerably affected not only by moduli of elasticity and volume fractions of particular materials constituting the component, but also by the internal geometry and structure of particular part (porosity distribution, actual distribution of constituents, etc.) and the geometrical shape of the part as a whole.

In the paper presented here, it is demonstrated how the variation of porosity distribution along the specimen cross section affects the value of effective flexural modulus measured by means of the dynamic resonant method [1]. Experimental results are compared qualitatively to effective flexural moduli calculated for simple theoretical models of real specimens. The characteristic behaviour of experimental moduli agrees with that of theoretical ones.

2. Experimental

Specimens with a quasilayered distribution of porosity were prepared from an iron powder by pressing and sintering with subsequent hammer forging. As a basis we used a water atomized iron powder WPL-200, produced by means of Mannesmann equipment at ZVL-METALSINT, a.s., Dolný Kubín, Slovakia. The particles of the powder are of approximately equiaxed type with the characteristic surface morphology.

To obtain the specimens with low and inhomogeneously distributed porosity, the following technological procedure was used: Samples were compacted at 600 MPa to 4 different heights (12, 10, 8, 7 mm). The compacts were then sintered for 2 hrs at 1120 °C in a retort silit furnace, the atmosphere being cracked ammonia (75 % H_2 + 25 % N_2). The dew point of the atmosphere was −20 °C. After sintering, the samples were further densified by hammer forging at 1100 °C in hydrogen atmosphere to final height of 6 mm. The densified semi-products were subsequently shaped to the form suitable for testing. The final size of the resulting bars being tested was 6 × 6 × 90 mm. The density of these rectangular bars was determined by measuring the dimensions and by weighing. The porosity obtained was in the range from 2.4 to 7.6 %.

Metallographic study revealed that the above described technological procedure led to nonhomogeneous porosity distribution within the samples (Fig. 1). The porosity was varying along the pressing (and forging) direction and its value decreased from the surface to the bulk of specimens. Such quasilayered bars were used for measuring the flexural moduli. After the measurements of moduli on intact bars had been completed, the porous surface layers were ground off and the testing bars were shaped to the new final size 3 × 3 × 90 mm. We obtained a set of specimens

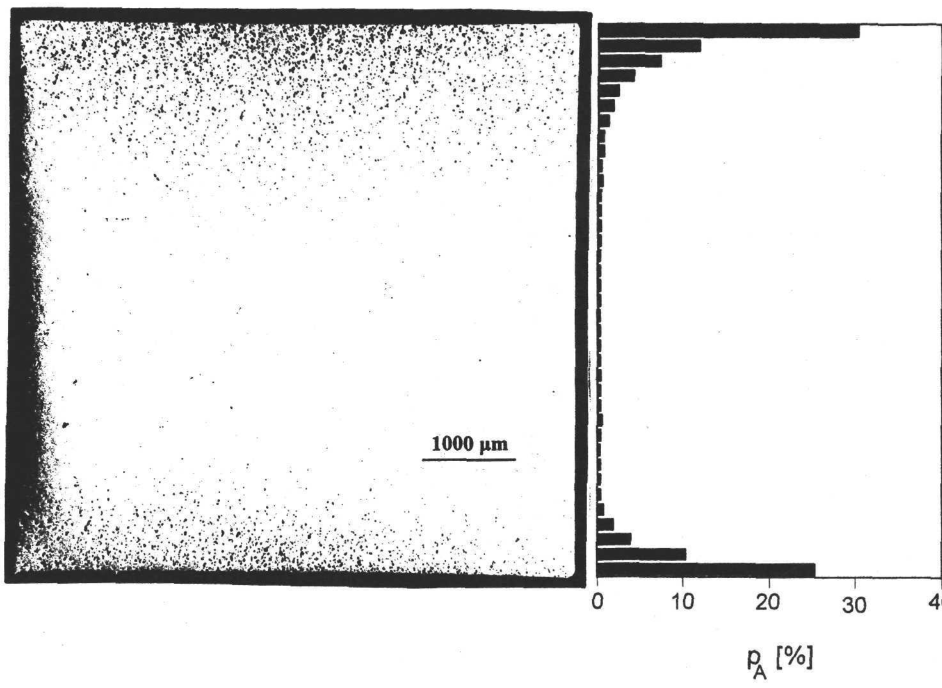

Fig. 1. Polished cross section of a bar of total porosity 2.7 %, made of a sintered iron and the areal fraction p_A that the pores occupy from the particular bands of the cross section presented. The areal fraction was determined by means of the image analysis. In the figure, pressing (and forging) direction is the vertical one.

with porosities that were lower than the porosities of original (not ground) samples. The differences in porosities of a bar before and after grinding were usually about 2 %. The distribution of porosity in ground samples was more homogeneous than in intact ones. These modified bars were again used for measuring the moduli.

The dynamic resonant method [1] was used for determining the effective flexural modulus. The frequency of the specimen natural vibrations was measured employing the apparatus Gringo Sonic MKS "Industrial" at University of Vienna. The frequency of flexural fundamental mode for the bar with free ends was used for evaluating the modulus. For a uniform, homogeneous and isotropic bar the flexural modulus is the same as the Young's modulus of the bar material. Since our specimens were not isotropic and homogeneous, the modulus measured was an effective modulus and differences in the values obtained parallel and perpendicular, respectively, to the pressing direction were expected. In fact it turned out that the E values determined by means of flexural vibration with bending plane perpendicular to quasilayers were lower than those determined by means of vibration with bending plane parallel to quasilayers (Fig. 2a). For specimens with (at least partially) removed surface layers, the differences between these two moduli were much smaller than for original bars, and for some specimens this difference even vanished (Fig. 2b).

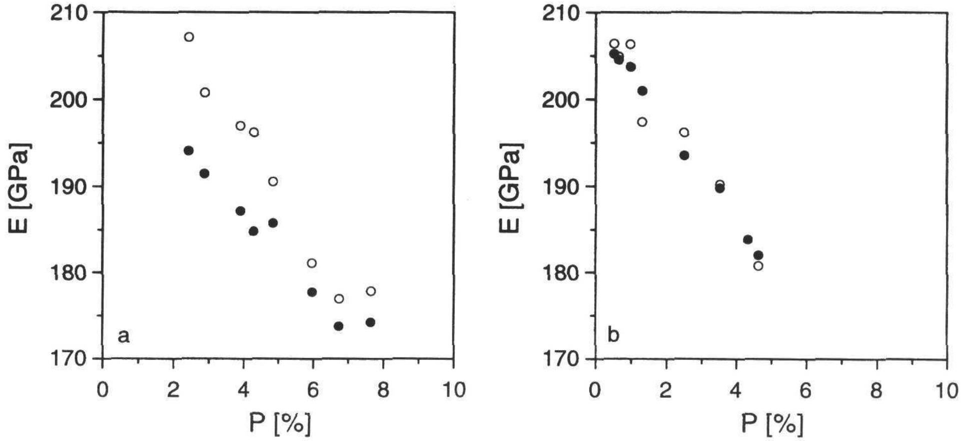

Fig. 2. Effective flexural modulus E as a function of total porosity P. Values presented were determined in an experimental way from the specimen flexural vibration with the bending plane perpendicular (•) and parallel (∘) to quasilayers for original samples (a) and for samples with removed surface layers (b).

3. Theory

To investigate the effect of quasilayered structure on effective flexural moduli in a theoretical way, the standard "dynamic-resonant-method" evaluating formula,

$$E^{\mathrm{fl}} = 0.94642 \frac{M}{WHL} L^4 \frac{f^2}{t^2}, \tag{1}$$

was used for determining the flexural modulus values E^{fl}. But in place of the experimentally measured resonant frequency, f, the theoretically calculated natural frequency of a quasilayered bar was used as an "input parameter". Expr. (1) stands for a rectangular bar undergoing free flexural vibration with free ends. M is the mass of the bar, H, W and L are height, width and length of the bar, respectively. t represents the specimen cross-sectional dimension in the direction of vibration, i.e., $t = H$ if the bending plane is parallel to HL-plane, or $t = W$ if the bending plane is parallel to WL-plane.

Corresponding theoretical expressions are published in our papers [2, 3]. Here we only briefly sketch the method of derivation and state the resultant relations. A rectangular bar of height H, width W and length L was considered (Fig. 3). Properties of material of the bar were assumed as varying only in the direction of the bar height. Vibration frequencies are obtained by solving the corresponding equation of motion. The equation of motion is derived by means of the Hamilton's principle of minimal action. Lagrange's function, occurring in the expression for

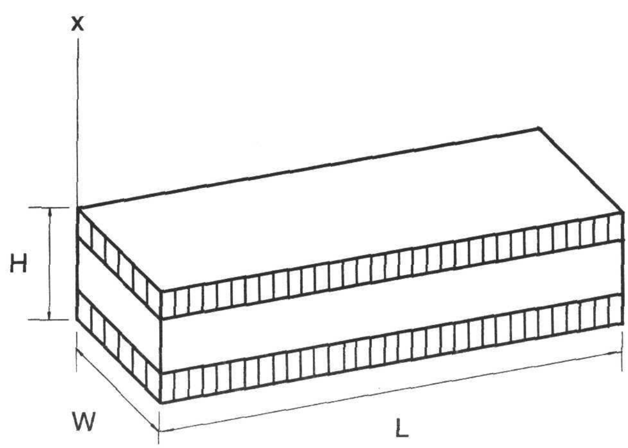

Fig. 3. Schematic sketch of a bar considered as a theoretical model of real quasilayered bars.

action, consists of the kinetic and potential (elastic) energies of a deformed bar. The required elastic energy is determined by means of the strain and stress tensor fields derived for a bent quasilayered bar under consideration. The geometry of deformation of material fibres and bar planar cross sections in a bent quasilayered bar is similar to the geometry of deformation in a homogeneous bar. Therefore, the strain tensor is qualitatively similar to the strain tensor in a homogeneous bar. The stress tensor is determined on the basis of the Hooke's law by means of the above mentioned strain tensor and the nonhomogeneous distribution of the material Young's modulus values along the cross section. The resultant elastic energy of a bent quasilayered bar, if expressed by means of the neutral-fibre curvature, differs from the elastic energy of a homogeneous bar only by a pre-factor – the flexural rigidity of the bar. The kinetic energy of a quasilayered bar, if expressed by means of the velocity of the neutral-fibre transverse motion, differs from that of a homogeneous bar by the pre-factor – mass of the bar per unit length. These differences lead to the analogous changes of corresponding quantities in the equation of motion and consequently in the relation for frequency of a homogeneous bar when they are rederived for the quasilayered bar.

Substituting theoretical frequencies into the formula (1) used for evaluating the "experimental" moduli, the following relations for effective moduli are obtained:

$$E_{\perp}^{\mathrm{fl}} = \frac{12}{H^3}\left[\int_0^H E(x)x^2\mathrm{d}x - \left(\int_0^H E(x)x\mathrm{d}x\right)^2 \left(\int_0^H E(x)\mathrm{d}x\right)^{-1}\right] \tag{2}$$

for vibration in the HL-plane, i.e. bending occurring perpendicular to quasilayers, and

$$E_{\parallel}^{\mathrm{fl}} = \frac{1}{H}\int_0^H E(x)\mathrm{d}x \tag{3}$$

for vibration in the WL-plane, i.e. bending occurring parallel to quasilayers. $E(x)$ represents the materials' Young's modulus that can vary along the height of the bar.

To calculate theoretical effective moduli (2) and (3), we need to find (or estimate) the Young's modulus $E(x)$ that properly simulates the relevant properties of real bars and that can be substituted to relations (2), (3).

According to relations among bar cross-sectional dimensions, pore sizes, and the rate of change of porosity along the cross section, we suppose that the real porous material in the vicinity of each point of the bar can be treated as the equivalent poreless material with an effective Young's modulus $E(x)$. In addition, we assume that within the porosity range under consideration the Young's modulus of equivalent material decreases linearly with increasing porosity of real material. So, for the Young's modulus $E(x)$ at a given point of the bar we have

$$E(x) = E_0 + \frac{\mathrm{d}E}{\mathrm{d}P} p(x), \tag{4}$$

where $p(x)$ represents the local porosity. E_0 and $\mathrm{d}E/\mathrm{d}P$ are the Young's modulus value of poreless material and the rate of Young's modulus decrease, respectively. Taking into account the material used for preparation of the samples and experimental data obtained, we were using the values 212 GPa for E_0 and −5 GPa per one percent of porosity for $\mathrm{d}E/\mathrm{d}P$.

The proper choice of local porosity $p(x)$ represents another step in determining the theoretical effective moduli. The porosity of real quasilayered bars were often distributed asymmetrically. Nevertheless, we chose a symmetric function $p(x)$, as (i) such distribution is sufficient for a qualitative demonstration of the characteristic behaviour of effective flexural moduli, and (ii) calculations are easier. Of course, if we would like to compare experimental and theoretical results also quantitatively, more complex porosity distribution should be chosen.

So as a simple theoretical model of our quasilayered bars, we considered a bar with local porosity $p(x)$ that increases according to power law with distance from the bar centre. There are several ways how to express this power-law dependence. The following relation was used in our calculations

$$p(x) = p_\mathrm{B} + (n+1)(P_\mathrm{H} - p_\mathrm{B}) \left(\frac{2x - H}{H}\right)^n, \quad n \text{ is even integer.} \tag{5}$$

p_B represents the porosity in the centre of the bar. Total porosity, P_H, is determined as $P_\mathrm{H} = \frac{1}{H}\int_0^H p(x)\mathrm{d}x$. The removal of surface layers is simulated by restricting the values available for x, keeping at the same time values $p_\mathrm{B}, P_\mathrm{H}$ and H constant and corresponding to the original bar. That is, for original bar $0 \le x \le H$, for ground bar $H/4 \le x \le 3H/4$. Of course, the total porosity of a "ground" sample will be lower than P_H. Fig. 4 presents the distributions of local porosity along the height of a model bar for $n = 2$ and $n = 4$.

Some of theoretical results for effective flexural moduli are demonstrated in Fig. 5. Quantities presented are effective flexural moduli calculated for a bar with local porosity increasing as the second (a, b) and fourth (c, d) powers of the distance from the bar centre. Parameters determining porosity distribution (5) were chosen in such a way that the local porosity in the bar centre, p_B, was kept lower by 2 % than the total porosity of the original (intact) bar, i.e., $p_\mathrm{B} = P$-2 %. Moduli

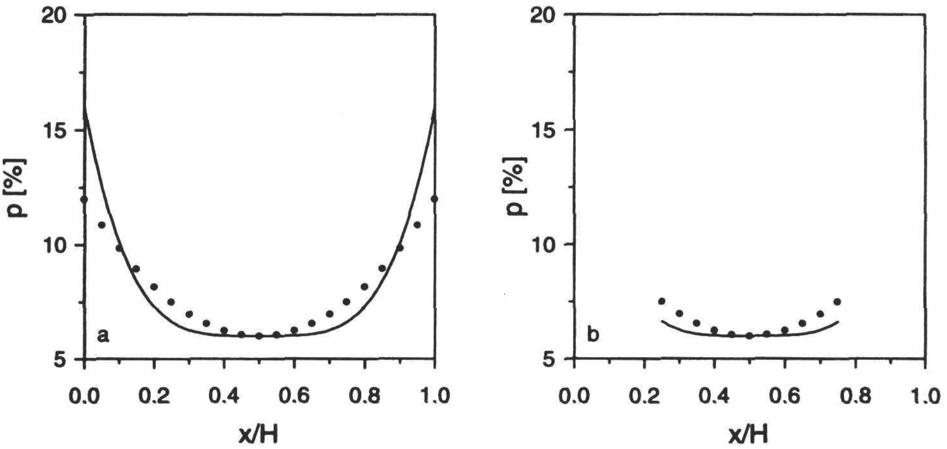

Fig. 4. Local porosity p as a function of position along the bar height, x/H. The curves presented are for quadratic (dotted lines) and quartic (solid lines) dependences, for original bars of full height ($0 \leq x/H \leq 1$) (a) and for "ground" bars of half height ($0.25 \leq x/H \leq 0.75$) (b). For all curves, parameters $P_H = 8$ % and $p_B = 6$ % were used.

were calculated for bars of original height (Fig. 5a,c) as well as for bars of half height (Fig. 5b,d) (a theoretical model for bars with ground off the surface layers). We can see that modulus values determined by means of vibration perpendicular to quasilayers are lower than those determined by means of vibration parallel to quasilayers.

With increasing exponent n, the difference between $E_{\parallel}^{\text{fl}}$ and $E_{\perp}^{\text{fl}}$ increases for original bars (Fig. 5a,c) and decreases for "ground" bars (Fig. 5b,d). It is due to the fact that with increasing exponent n, the distribution (5) of local porosity $p(x)$ becomes much flatter in the central region of the bar and sharply increases in thin surface regions (Fig. 4a). The higher the exponent n is, the thinner these regions are. So, the distribution of porosity in the original bar becomes more heterogeneous, with higher gradient of porosity concentrated within thinner regions near surfaces. But if we take into account only a bar central region (after "grinding" the original bar), the distribution $p(x)$ becomes more homogeneous with increasing n (Fig. 4b). Therefore, for the "ground" bars of half heights, difference between $E_{\parallel}^{\text{fl}}$ and $E_{\perp}^{\text{fl}}$ decreases with increasing n and for sufficiently high n this difference even vanished.

4. Discussion and conclusions

In this paper, the effect of quasilayered distribution of porosity in testing bars

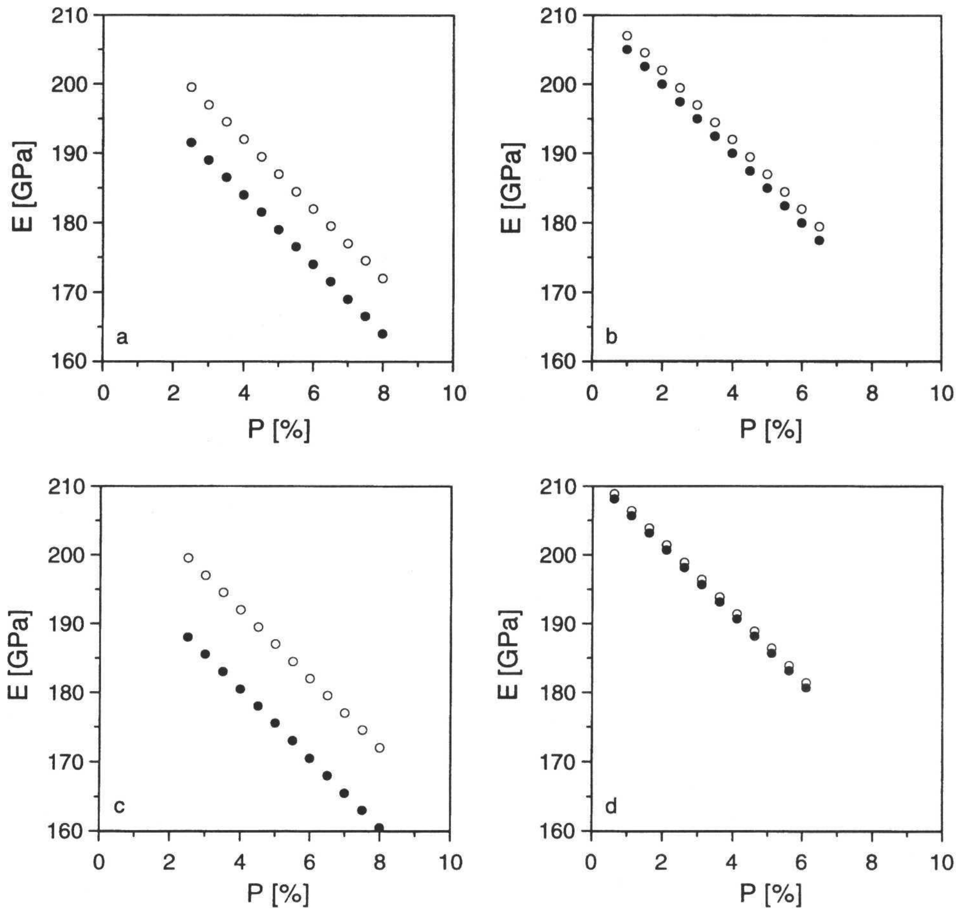

Fig. 5. Effective flexural modulus E as a function of total porosity P. Values presented were determined in a theoretical way for quasilayered bars with quadratic (a,b) and quartic (c,d) dependence of local porosity on the distance from the bar centre. Moduli were calculated by means of frequencies of flexural vibration with bending plane perpendicular (•) and parallel (○) to quasilayers. The bars of original height (a,c) as well as bars of half height (b,d) were considered.

on the measured flexural moduli is demonstrated both in experimental and theoretical ways. Rectangular bar-shaped samples were prepared from iron powder. The specimens possessed a quasilayered structure consisting of surface regions with higher porosity and relatively less porous core (Fig. 1). Flexural moduli measured by means of flexural vibration parallel to quasilayers were higher than those measured by means of vibrations perpendicular to quasilayers (Fig. 2a). When the porous regions at the surface were more or less removed, the difference between these two moduli became much smaller, and sometimes even vanished (Fig. 2b). This effect of surface regions with less stiffness on the effective flexural moduli was demonstrated also theoretically by calculating the relevant moduli for model bars simulating the real specimens. Theoretical results confirmed that for quasilayered bars with core stiffer than surface regions the "parallel" flexural modulus is usually higher than the "perpendicular" one. But as the difference between properties of core and surface decreases (surface layers are successively removed), the difference between these two moduli decreases, too (Fig. 5).

Such behaviour of flexural moduli is caused by the fact that for rectangular bar-shaped samples the flexural rigidity (which actually determines the specimen bending properties [4]) is mostly affected by stiffness of regions located near the surfaces perpendicular to bending plane. So if our quasilayered bars are bent in the plane perpendicular to quasilayers, the relevant regions are surface porous layers with lowered stiffness due to higher porosity. On the other hand, for bending in the plane parallel to quasilayers, relevant surface regions contain not only a part of porous layers but also a part of stiffer, less porous core layer. So one can expect that for our bars the effective flexural modulus, which is, by definition, the Young's modulus of a hypothetical homogeneous bar of the same size, shape and flexural rigidity as the actual quasilayered one, will be lower for bending in the plane perpendicular to quasilayers than for bending in the plane parallel to quasilayers. In the opposite case, if the quasilayered bars would consist of stiffer surface layers and a core region of less stiffness, the bending in the plane perpendicular to quasilayers would give higher value of the effective flexural modulus than bending in the plane parallel to quasilayers. Anyway, as the samples become more homogeneous, that is, difference between properties of surface and core decreases, difference between the two moduli decreases, too.

To compare the theory and experiment quantitatively, we need detailed information on the distribution, sizes and shapes of pores within real samples. Such complex data should be only determined by combination of several experimental methods [5]. It represents the aim of our further investigation.

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